Mean, variance and standard deviation for continuous models

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

1. For density $3x^2$ on $[0, 1]$, find $\mathrm{E}(X)$. [2 marks]

$\int_0^1 3x^3\,dx = \frac{3}{4}$.

Answer: $\frac{3}{4}$.

2. For density $3x^2$ on $[0, 1]$, find $\mathrm{E}(X^2)$ and variance. [3 marks]

$\int_0^1 3x^4\,dx = \frac{3}{5}$.

$\frac{3}{5} - (\frac{3}{4})^2 = \frac{3}{80}$.

Answer: $\frac{3}{5}$ and $\frac{3}{80}$.

3. Find the mean and variance of a uniform variable on $[0, 6]$. [2 marks]

Use the midpoint and $\frac{6^2}{12} = 3$.

Answer: Mean 3; variance 3.

4. For $\mathrm{E}(X) = \frac{2}{3}$ and $\mathrm{Var}(X) = \frac{1}{18}$, find the mean and variance of $5 - 2X$. [2 marks]

$5 - 2(\frac{2}{3}) = \frac{11}{3}$; $\frac{(-2)^2}{18} = \frac{2}{9}$.

Answer: $\frac{11}{3}$ and $\frac{2}{9}$.

5. A length has variance 0.04 square metres. Find its standard deviation. [2 marks]

Take the positive square root and use the original unit.

Answer: 0.2 m.

6. For density $\frac{1}{2}$ on $[-1, 1]$, find mean and variance. [3 marks]

Symmetry gives mean zero.

$\mathrm{E}(X^2) = \frac{1}{2}\int_{-1}^{1} x^2\,dx = \frac{1}{3}$.

Answer: 0 and $\frac{1}{3}$.