Cumulative distribution functions

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

1. For CDF x^2 on $[0, 1]$, find $P(0.4 < X < 0.8)$. [2 marks]

$0.8^2 - 0.4^2 = 0.48$.

Answer: 0.48.

2. Give the CDF of density $3x^2$ on $[0, 1]$ over the whole real line. [3 marks]

Integrate $3t^2$ from 0 to x inside the support; use outer cumulative limits.

Answer: 0 below 0; x^3 on $[0, 1]$; 1 above 1.

3. Find the 90th percentile for CDF x^2 on $[0, 1]$. [2 marks]

Solve $q^2 = 0.9$ in the support.

Answer: $\sqrt{0.9} \approx 0.9487$.

4. Can a CDF decrease from 0.6 to 0.5 as x increases? Explain. [2 marks]

An increasing upper bound cannot remove already accumulated probability.

Answer: No.

5. X is uniform on $[4, 10]$. Find $F(2)$ and $F(12)$. [2 marks]

2 is below the support; 12 is above it.

Answer: 0 and 1.

6. For CDF $1 - (1 - x)^2$ on $[0, 1]$, find the density and median. [3 marks]

Differentiate the CDF.

Solve $(1 - m)^2 = \frac{1}{2}$ and retain the root in $[0, 1]$.

Answer: $f(x) = 2(1 - x)$; median $1 - \frac{1}{\sqrt{2}}$.