Probability density functions and total area

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Is $f(x) = 0.8$ on $[0, 2]$ a density? Explain. [2 marks]

Total area $0.8(2) = 1.6$, not one.

Answer: No.

2. Find k if $f(x) = kx$ on $[0, 3]$ is a density. [2 marks]

$\int_0^3 kx\,dx = \frac{9k}{2} = 1.$

Answer: $k = \frac{2}{9}$.

3. Check both conditions for $f(x) = 3x^2$ on $[0, 1]$. [2 marks]

$3x^2 \geq 0$ and $\int_0^1 3x^2\,dx = 1.$

Answer: It is valid.

4. For any continuous density, find $P(X = 0.7)$. [2 marks]

One point has zero width and zero area.

Answer: 0.

5. A nonnegative triangular density has width 1 and height 2. Is its area correct? [2 marks]

$\frac{1}{2}(1)(2) = 1.$

Answer: Yes.

6. A function has integral one but is negative on part of its support. Is it a density? [2 marks]

Densities must be nonnegative on their support.

Answer: No.