# Discrete distributions - Applications and reasoning test

Name: _____ Date: _____

10 questions | 72 marks | 75 minutes

An integrated paper requiring distribution construction, nonlinear costs, model criticism and justified decisions.

Use a graphics calculator or appropriate digital tool for labelled tasks.

**1.** A count $X$ takes values 0, 1 and 3 with probabilities $a$, $b$ and 0.2. Its expected value is 1.1. [7 marks]

**(a)** Find $a$ and $b$. [3 marks]

**(b)** Find $\mathrm{Var}(X)$. [2 marks]

**(c)** Find $P(X \leq 1)$ and state whether observing 3 is impossible. [2 marks]

**2.** An integer-valued $X$ has $F(0)=a$, $F(1)=3a$, $F(2)=1-a$ and $F(3)=1$. No other values occur. Also $P(X=1)=0.4$. [7 marks]

**(a)** Find $a$ and the probability table. [3 marks]

**(b)** Find the mean and variance. [2 marks]

**(c)** Find $P(1<X\leq 3)$. [2 marks]

**3.** A game prize $X$ is 0 dollars, 3 dollars or 12 dollars with probabilities 0.6, 0.3 and 0.1. [7 marks]

**(a)** Find its fair entry fee and prize variance. [3 marks]

**(b)** A promotion gives twice the prize but charges 5 dollars. Find expected net gain and its variance. [2 marks]

**(c)** Explain why doubling prizes does not necessarily benefit the player on average. [2 marks]

**4.** A policy has a claim of 1000 dollars with probability 0.02 and no claim otherwise. Administration costs 8 dollars per policy. The premium $c$ must be a whole number of dollars. [8 marks]

**(a)** Find the expected claim and the smallest premium giving expected profit at least 15 dollars. [3 marks]

**(b)** At that premium, find the probability of a negative profit on a single policy. [2 marks]

**(c)** Find the standard deviation of single-policy profit. [3 marks]

**5.** A usage count $X$ is 0, 1 or 2 with probabilities 0.5, 0.3 and 0.2. A nonlinear cost is $C = 5 + 2X + X^2$ dollars. [7 marks]

**(a)** Construct the distribution of $C$. [2 marks]

**(b)** Find $E(C)$ and $\mathrm{Var}(C)$. [3 marks]

**(c)** Why should the affine variance rule not be applied directly to the whole expression for $C$ ? [2 marks]

**6.** Independent daily demands are $A = 0$ or 1 with probabilities 0.4 and 0.6, and $B = 0$ or 2 with probabilities 0.7 and 0.3. Set $T = A + B$. [7 marks]

**(a)** Find the distribution of $T$. [3 marks]

**(b)** Find $E(T)$ and $\mathrm{Var}(T)$. [2 marks]

**(c)** Would the probability table in (a) still follow from the marginal tables if independence were removed? [2 marks]

**7.** Use the distribution $T = 0, 1, 2, 3$ with probabilities 0.28, 0.42, 0.12, 0.18. A shortage occurs when $T \geq 2$. Assume seven independent days follow this same distribution. [6 marks]

**(a)** Define a shortage indicator and give its mean and variance. [2 marks]

**(b)** State the distribution, mean and variance of the number $S$ of shortage days. [2 marks]

**(c)** Find $P(S \geq 3)$. [2 marks]

**8.** Each batch has one shared environment: equally likely low-risk or high-risk. Conditional on that environment, two items fail independently with probabilities 0.2 or 0.8 respectively. $X$ counts failed items. [8 marks]

**(a)** Find each item's unconditional failure probability. [2 marks]

**(b)** Construct the unconditional distribution of $X$. [3 marks]

**(c)** Compare its variance with $\text{Bin}(2, 0.5)$ and identify the failed assumption. [3 marks]

**9.** For independent trials with success probability 0.1, define $X_n$ as the success count in $n$ trials. A design requires $P(X_n \geq 5) \leq 0.05$. [8 marks]

**(a)** Evaluate the tail probability for $n = 19$, 20 and 21. [3 marks]

**(b)** Use the three values to conjecture how $P(X_n \geq 5)$ changes as $n$ increases with $p$ fixed. State the largest acceptable $n$ among 19, 20 and 21. [2 marks]

**(c)** Justify your conjecture using the relationship between counts in $n$ and $n+1$ trials, with $p$ fixed. [3 marks]

**10.** A binomial model for a daily count has exact mean 6 and variance 4.5. An alert is raised for a count of at least 10. [7 marks]

**(a)** Find $n$ and $p$. [3 marks]

**(b)** Find the probability of an alert. [2 marks]

**(c)** If the true success probability changes to 0.2, may the old alert probability be used unchanged? Explain. [2 marks]