Discrete distributions - Practice test 5

Name: _____ Date: _____

10 questions | 65 marks | 75 minutes

Connect cumulative thresholds, nonlinear transformations, costs and repeated sampling without losing the model assumptions.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. A fair spinner has five equal regions labelled 0, 0, 1, 3 and 3. X is the label on one spin. [6 marks]

(a) Construct the probability distribution of X. [2 marks]

(b) Find $E(X)$ and $\text{Var}(X)$. [2 marks]

(c) Find $P(X \geq 1)$. [2 marks]

2. X takes 0, 1, 2 and 3 with probabilities 0.1, 0.25, 0.4 and 0.25. [6 marks]

(a) Give its cumulative probabilities at the four values. [2 marks]

(b) Find the smallest value x for which $P(X \leq x) \geq 0.5$. [2 marks]

(c) Find the smallest x for which $P(X \leq x) \geq 0.9$. [2 marks]

3. A traveller chooses protection plan A costing a fixed 10 dollars, or plan B costing 4 dollars plus an excess of 50 dollars if a claim occurs. Claim probability is 0.1; there can be at most one claim. [6 marks]

(a) Find the expected cost of each plan. [2 marks]

(b) Find the variance of cost for each plan. [2 marks]

(c) Explain why choosing A need not be irrational despite its higher expected cost. [2 marks]

4. X takes 0, 1 and 2 with probabilities a, $1-2a$ and a. [7 marks]

(a) Show that $E(X)=1$ for every valid a. [2 marks]

(b) Given $\mathrm{Var}(X)=0.4$, find a and all probabilities. [3 marks]

(c) Could the same family have variance 1.2? Explain. [2 marks]

5. X is 0, 1 or 2 with probabilities 0.5, 0.25 and 0.25. Let $Y = X^2$. [7 marks]

(a) Find $E(X)$ and $\text{Var}(X)$. [2 marks]

(b) Find $E(Y)$ and $\text{Var}(Y)$ directly. [3 marks]

(c) Why can the linear variance rule not be used with $a = 2$? [2 marks]

6. Sixteen independent copies of X each have mean 5 and variance 4. Let T be their sum and A their average. [7 marks]

(a) Find $E(T)$ and $\mathrm{Var}(T)$. [2 marks]

(b) Find $E(A)$, $\mathrm{Var}(A)$ and $\mathrm{SD}(A)$. [3 marks]

(c) Explain why the average has smaller spread without a smaller mean. [2 marks]

7. The joint probabilities of indicators B and C are shown. [7 marks]

Joint probabilities

B	C	Probability
0	0	0.4
1	0	0.2
0	1	0.1
1	1	0.3

(a) Find $P(B=1)$ and $P(C=1)$. [2 marks]

(b) Find the distribution and variance of $T=B+C$. [3 marks]

(c) Test independence and compare with adding their separate variances. [2 marks]

8. In 20 tosses of a new coin, 10 heads and 10 tails are observed. [6 marks]

(a) Give the relative-frequency estimate of its head probability. [2 marks]

(b) Does this result prove future tosses are independent with $p = 0.5$? Explain. [2 marks]

(c) State a conditional binomial model for heads in the next 12 tosses if those assumptions are adopted. [2 marks]

9. $X \sim \text{Bin}(25, 0.12)$. [6 marks]

(a) Find $P(2 \leq X \leq 5)$, to four decimal places. [2 marks]

(b) Find $P(X \geq 6)$. [2 marks]

(c) Out of 100 comparable samples, how many are expected to have at least six successes? [2 marks]

10. An independent trial succeeds with probability 0.05. A team needs at least one success with probability at least 0.95. [7 marks]

(a) Form an inequality for the required trial count n. [2 marks]

(b) Find the smallest n and verify the adjacent boundary. [3 marks]

(c) Would knowing that the mean number of successes is above 1 be enough to guarantee the target? [2 marks]