Discrete distributions - Practice test 4

Name: _____ Date: _____

10 questions | 64 marks | 70 minutes

Explain distribution identities, distinguish dependent indicators and solve a minimum-trial problem.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. X takes values 0, 1, 2 and 3, with $P(X = x) = k(x + 1)$. [6 marks]

(a) Find k and $P(X \geq 2)$. [2 marks]

(b) Find $E(X)$ and $\text{Var}(X)$. [2 marks]

(c) Find $F(1.5)$. [2 marks]

2. X takes values 0, 1, 2 and 3. Its cumulative values are $F(0)=0.15$, $F(1)=a$, $F(2)=0.8$ and $F(3)=1$. Also $P(X=1)=0.35$. [6 marks]

(a) Find a. [2 marks]

(b) Find $P(X=2)$. [2 marks]

(c) Find $P(X\geq 2)$ and explain the endpoint. [2 marks]

3. X takes -2, 0 and 2 with probabilities $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$. Let $Y = X^2$. [7 marks]

(a) Find $E(X)$, $E(X^2)$ and $\text{Var}(X)$. [2 marks]

(b) Construct the distribution of Y and find its variance. [3 marks]

(c) Explain why $E(X^2)$ is not $[E(X)]^2$ here. [2 marks]

4. D is the score on a fair six-sided die. [6 marks]

(a) Calculate $E(D^2)$. [2 marks]

(b) Use $E(D) = 3.5$ to find $\text{Var}(D)$. [2 marks]

(c) For two independent dice, find the variance of their total. [2 marks]

5. X has mean $\mu = 30$ and standard deviation $\sigma = 6$. Define $Z = \frac{X - 30}{6}$. [6 marks]

(a) Find $E(Z)$ and $\text{Var}(Z)$. [2 marks]

(b) Express the event $Z > 2$ as an event involving X. [2 marks]

(c) Can $P(Z > 2)$ be obtained from the supplied mean and SD alone? [2 marks]

6. Three independent indicators have success probabilities 0.1, 0.4 and 0.8. T is their total. [6 marks]

(a) Find its mean and variance. [2 marks]

(b) Find $P(T=0)$ and $P(T=3)$. [2 marks]

(c) Explain why matching the mean with $\text{Bin}(3, \frac{1.3}{3})$ does not establish that model. [2 marks]

7. A fair coin defines $B = 1$ for heads. Let $C = 1 - B$ and $T = B + C$. [6 marks]

(a) Find $E(T)$ and $\text{Var}(T)$ directly. [2 marks]

(b) Calculate what an incorrect independence assumption would give for $\text{Var}(T)$. [2 marks]

(c) Explain the dependence. [2 marks]

8. $X \sim \text{Bin}(2, p)$, where $0 < p < 1$, and $P(X = 2) = 9P(X = 0)$. [7 marks]

(a) Form and solve an equation for p. [3 marks]

(b) Find $P(X = 1)$. [2 marks]

(c) Find the mean and variance. [2 marks]

9. Independent trials have success probability 0.2. A planner wants the probability of at least one success to be at least 0.95. [7 marks]

(a) Write the required inequality in n. [2 marks]

(b) Find the smallest integer n. [3 marks]

(c) Verify the boundary using $n-1$ and n. [2 marks]

10. Game A charges 2 dollars and pays 10 dollars with probability 0.2, otherwise 0 dollars. Game B charges 2 dollars and always returns 2 dollars. A player can choose either. [7 marks]

(a) Compare expected net gains. [2 marks]

(b) Compare net-gain variances. [2 marks]

(c) For ten independent plays of A, find the probability of a strictly positive total net gain. [3 marks]