Discrete distributions - Practice test 3

Name: _____ Date: _____

10 questions | 62 marks | 70 minutes

Apply the models to service times, insurance, tariffs and capacity decisions, with explicit assumptions.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. A waiting time X is 1, 2 or 4 minutes with probabilities 0.3, 0.5 and 0.2. [6 marks]

(a) Find its expected waiting time. [2 marks]

(b) Find its variance and standard deviation. [2 marks]

(c) What is $P(X > 2)$? [2 marks]

2. A team records 100 daily counts: 30 days have count 0, 50 have count 1 and 20 have count 2. [6 marks]

(a) Use relative frequencies to estimate a distribution for one daily count. [2 marks]

(b) Find the mean under this estimated model. [2 marks]

(c) Does the model guarantee exactly 20 count-2 days in the next 100 days? Explain. [2 marks]

3. One insurance policy has claim X equal to 0 dollars, 500 dollars or 2000 dollars with probabilities 0.96, 0.03 and 0.01. The premium is 50 dollars. [6 marks]

(a) Find the expected claim. [2 marks]

(b) Find expected profit per policy before administrative costs. [2 marks]

(c) Allowing a fixed administrative cost of 5 dollars per policy, find expected profit across 200 policies. [2 marks]

4. A daily repair cost X, in tens of dollars, is 0, 2 or 6 with probabilities 0.5, 0.4 and 0.1. [6 marks]

(a) Find its mean and variance in the stated units. [2 marks]

(b) Find the standard deviation in dollars. [2 marks]

(c) A fixed call-out fee of 50 dollars is added. What happens to the dollar standard deviation? [2 marks]

5. A delivery tariff is $T = 4 + 2.5X$ dollars. The daily distance X has mean 3.2 km and variance 1.5 km². [6 marks]

(a) Find $E(T)$. [2 marks]

(b) Find $\mathrm{Var}(T)$ and $\mathrm{SD}(T)$. [2 marks]

(c) Explain why an increase of 1 dollar in the fixed tariff leaves $\mathrm{Var}(T)$ unchanged. [2 marks]

6. Independent service times A and B have means 3 and 4 minutes and variances 0.25 and 1 min². [6 marks]

(a) Find the mean and variance of $A - B$. [2 marks]

(b) Does the negative mean prove that $A < B$ on every occasion? [2 marks]

(c) Can $P(A < B)$ be calculated uniquely from the supplied moments alone? Explain. [2 marks]

7. A daily count X is 0, 1, 2 or 3 with probabilities 0.55, 0.25, 0.15 and 0.05. A flag B is 1 when $X \geq 2$ and 0 otherwise. [6 marks]

(a) Find $P(B = 1)$, $E(B)$ and $\mathrm{Var}(B)$. [2 marks]

(b) For 40 independent comparable days, find the mean and variance of the number of flags. [2 marks]

(c) Name a plausible reason independence might fail. [2 marks]

8. A day is equally likely to be quiet or busy. Conditional on the day type, two customer outcomes are independent. Each customer buys with probability 0.2 on a quiet day and 0.6 on a busy day. [7 marks]

(a) Find one customer's unconditional purchase probability. [2 marks]

(b) Find the unconditional probability that both buy. [3 marks]

(c) Explain why $\text{Bin}(2, 0.4)$ is not exact for the purchase count. [2 marks]

9. An independent-item fault model gives a fault probability of 0.08. Each batch contains 12 items. [6 marks]

(a) Find the probability of at most one faulty item. [2 marks]

(b) Find the probability of at least two faulty items. [2 marks]

(c) Among 300 batches, find the expected number containing at least two faults. [2 marks]

10. Twenty independent customers each require a service with probability 0.1. Capacity c is exceeded when more than c customers require it. [7 marks]

(a) Find the exceedance probabilities for capacities 3 and 4. [3 marks]

(b) Choose the smaller of these capacities that keeps exceedance probability below 0.05. [2 marks]

(c) Give one limitation of the decision. [2 marks]