# Discrete distributions - Practice test 1

Name: _____ Date: _____

10 questions | 58 marks | 70 minutes

Build confidence with tables, cumulative probabilities, summary measures and an ordinary binomial model.

Use a graphics calculator or appropriate digital tool for labelled tasks.

**1.** $X$ has the probability distribution shown. [6 marks]

Distribution of $X$

| $x$ | 0 | 1 | 2 | 4 |
|---|---|---|---|---|
| $P(X=x)$ | 0.2 | 0.3 | $k$ | 0.1 |

**(a)** Find $k$. [2 marks]

**(b)** Calculate $E(X)$. [2 marks]

**(c)** Find $P(X \geq 2)$. [2 marks]

**2.** $X$ is integer-valued and has $F(0)=0.1$, $F(1)=0.4$, $F(2)=0.75$ and $F(3)=1$. It takes no other values. [6 marks]

**(a)** Find $P(X=2)$. [2 marks]

**(b)** Find $P(1\leq X\leq 2)$. [2 marks]

**(c)** Find $P(X>1)$. [2 marks]

**3.** A game pays 0 dollars, 5 dollars or 20 dollars with probabilities 0.6, 0.3 and 0.1. The entry fee is 4 dollars. [6 marks]

**(a)** Find the expected gross prize. [2 marks]

**(b)** Find the fair fee and expected net gain at the actual fee. [2 marks]

**(c)** Find expected total net gain over 200 plays. [2 marks]

**4.** $X$ takes values $-1$, 1 and 3 with probabilities $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$. [5 marks]

**(a)** Find $E(X)$ and $E(X^2)$. [2 marks]

**(b)** Find $\text{Var}(X)$ and $\text{SD}(X)$. [2 marks]

**(c)** Explain why the variance is not expressed in the same units as $X$. [1 mark]

**5.** A score $X$ has mean 8 and variance 2. A conversion uses $Y = 3X - 5$. [6 marks]

**(a)** Find $E(Y)$. [2 marks]

**(b)** Find $\text{Var}(Y)$ and $\text{SD}(Y)$. [2 marks]

**(c)** If the subtraction of 5 is removed, what happens to the mean and variance? [2 marks]

**6.** Independent $X$ and $Y$ have means 4 and 3 and variances 5 and 2. [5 marks]

**(a)** Find the mean and variance of $2X + Y$. [2 marks]

**(b)** Find the mean and variance of $X - Y$. [2 marks]

**(c)** Which calculation in part (b) still works if independence is not known? [1 mark]

**7.** A check passes with probability 0.8. Let $B = 1$ for a failure and $B = 0$ for a pass. [6 marks]

**(a)** State the distribution and mean of $B$. [2 marks]

**(b)** Find $\mathrm{Var}(B)$. [2 marks]

**(c)** Find the mean and variance of the score $4B - 1$. [2 marks]

**8.** A biased coin has $P(\text{heads}) = 0.3$ and is tossed independently five times. $X$ counts tails. [5 marks]

**(a)** State a binomial model for $X$ and justify its parameters. [3 marks]

**(b)** A different process changes the coin bias after each toss. Explain which binomial condition may fail. [2 marks]

**9.** $X \sim \text{Bin}(8, 0.3)$. Give probabilities to four decimal places. [6 marks]

**(a)** Find $P(X = 2)$. [2 marks]

**(b)** Find $P(X \geq 2)$. [2 marks]

**(c)** State $E(X)$ and explain why it need not be a possible outcome. [2 marks]

**10.** A binomial variable has exact mean 12 and exact variance 9. [7 marks]

**(a)** Find $n$ and $p$. [3 marks]

**(b)** Find the probability of zero successes, using scientific notation if useful. [2 marks]

**(c)** Does the mean force each set of 48 trials to contain 12 successes? Explain. [2 marks]