Binomial assumptions

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. A fair die is rolled independently 8 times. X counts even scores. State n and p. [2 marks]

There are three even scores out of six.

The stated independent, fixed rolls supply the other binomial conditions.

Answer: $n = 8,\ p = \frac{1}{2}$.

2. Ten bulbs are tested independently, each with fault probability 0.03. State a model for the number of faults. [2 marks]

Success means a fault.

There are ten fixed independent tests with common success probability 0.03.

Answer: $X \sim \text{Bin}(10, 0.03)$.

3. Four cards are drawn without replacement from a standard deck. Explain why the heart count is not exactly $\text{Bin}(4, \frac{1}{4})$. [2 marks]

After drawing a heart there are 12 hearts among 51 cards.

After drawing a non-heart there are 13 hearts among 51; neither equals the unconditional first-draw probability.

Answer: The draws are dependent and the heart probability changes.

4. A player takes 20 independent shots, with success probability 0.8 for the first 10 and 0.6 for the last 10. Why is the total not exactly $\text{Bin}(20, 0.7)$? [2 marks]

The mean probability being 0.7 does not make every trial have probability 0.7.

The total is a sum of two counts with different success probabilities.

Answer: The success probability is not constant across trials.

5. An online poll stops when it has collected five positive responses. Identify the fixed-n problem. [2 marks]

The stopping condition fixes successes, not trials.

A binomial model requires a predetermined number of trials.

Answer: The number of people polled is random.

6. A factory samples 10 items without replacement from 100,000 items. Explain a possible approximate binomial model and a remaining limitation. [3 marks]

State $n = 10$ and estimate p from appropriate process information.

The draws are still not exactly independent, and clustering or a changing production process could undermine the approximation.

Answer: The sample fraction is tiny, so the fault probability changes little; independence is approximate.