Bernoulli trials

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. A coin lands heads with probability 0.55. Define a Bernoulli indicator for tails and give p. [2 marks]

The chosen success event is tails.

Its probability is $1-0.55$.

Answer: $B=1$ for tails, 0 for heads; $p=0.45$.

2. B is Bernoulli with $p=0.4$. Find $E(B)$, $\mathrm{Var}(B)$ and $\mathrm{SD}(B)$. [3 marks]

Use $E(B)=p$.

Use $\mathrm{Var}(B)=p(1-p)$ and take its square root.

Answer: $0.4;\ 0.24;\ \sqrt{0.24}\approx 0.4899$.

3. Let B be Bernoulli with $p=0.1$. A bonus is $Y=20B$ dollars. Find its mean and variance. [3 marks]

$E(Y)=20(0.1)=2$.

$\mathrm{Var}(Y)=20^2(0.1)(0.9)=36$.

Answer: Mean 2 dollars; variance 36 dollars².

4. If B is Bernoulli with $p=1$, describe B and find its variance. [2 marks]

Success is certain.

The formula $p(1-p)$ gives $1\times 0=0$.

Answer: B is always 1; variance 0.

5. A variable equals 2 or 7. Explain how a Bernoulli variable B can represent it. [2 marks]

If $B=0$ the expression is 2.

If $B=1$ it is 7; p is the probability of the outcome 7.

Answer: $X=2+5B$, with $B=1$ for the outcome 7.

6. Show that a Bernoulli variance cannot exceed $\frac{1}{4}$. [3 marks]

Complete the square in $p-p^2$.

A square is non-negative, with equality in the bound when $p=\frac{1}{2}$.

Answer: $p(1-p)=\frac{1}{4}-(p-\frac{1}{2})^2\leq\frac{1}{4}$.