Sums of independent variables

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. Independent X and Y have means 4, 7 and variances 1, 9. Find the mean and variance of $X+Y$. [2 marks]

$E(X+Y)=4+7$.

$\text{Var}(X+Y)=1+9$ by independence.

Answer: 11 and 10.

2. Using the same X and Y, find the mean and variance of $3X-2Y+5$. [3 marks]

Mean $=3(4)-2(7)+5=3$.

Variance $=9(1)+4(9)=45$.

Answer: Mean 3; variance 45.

3. Five independent copies of X each have mean 2 and variance 3. Find the mean and variance of their sum. [2 marks]

Add five means: $5(2)$.

Add five independent variances: $5(3)$.

Answer: Mean 10; variance 15.

4. For those five independent copies, find the mean and variance of their average. [3 marks]

The average is one fifth of the sum.

Its mean is $\frac{10}{5} = 2$; variance is $\frac{15}{25} = \frac{3}{5}$.

Answer: $2;\ \frac{3}{5}$.

5. Independent fair dice are rolled. Let X and Y be their scores. Find $P(X + Y = 5)$. [2 marks]

The ordered pairs are $(1, 4)$, $(2, 3)$, $(3, 2)$, $(4, 1)$.

Each has probability $\frac{1}{36}$, so the required probability is $\frac{4}{36}$.

Answer: $\frac{1}{9}$.

6. A supplier gives $E(X) = 8$, $E(Y) = 3$, $\mathrm{Var}(X) = 4$ and $\mathrm{Var}(Y) = 2$, but does not state independence. Which of $E(X + Y) = 11$ and $\mathrm{Var}(X + Y) = 6$ is justified? [3 marks]

Expectation is additive without independence.

The variance calculation also needs independence or suitable dependence information.

Answer: Only the mean statement is justified.