Effects of scale/shift

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

1. $E(X) = 5$ and $\text{Var}(X) = 4$. Find $E(2X + 3)$ and $\text{Var}(2X + 3)$. [2 marks]

$E(2X + 3) = 2(5) + 3.$

$\text{Var}(2X + 3) = 2^2(4).$

Answer: 13 and 16.

2. $E(X) = 8$ and $\text{SD}(X) = 3$. Find the mean and SD of $Y = 10 - X$. [2 marks]

$E(Y) = 10 - 8.$

The scale magnitude is 1, so the standard deviation stays 3.

Answer: Mean 2; SD 3.

3. A random length X in metres has mean 1.2 and variance 0.04. Convert both to centimetre-based quantities. [3 marks]

$Y = 100X.$

$E(Y) = 100(1.2) = 120;\ \text{Var}(Y) = 100^2(0.04) = 400.$

Answer: Mean 120 cm; variance 400 cm².

4. A score X has mean 50 and SD 10. Find mean and variance of $Z = \frac{X - 50}{10}$. [2 marks]

$E(Z) = \frac{50 - 50}{10} = 0.$

$\text{Var}(Z) = \frac{100}{100} = 1.$

Answer: Mean 0; variance 1.

5. X is -2 or 2 with equal probabilities. Find $E(X^2)$. Explain why squaring $E(X)$ gives the wrong answer. [3 marks]

X^2 always equals 4, so its mean is 4.

$E(X) = 0$, but a nonlinear transformation cannot be moved through expectation.

Answer: $E(X^2) = 4$; $[E(X)]^2 = 0$.

6. X is 1, 2 or 4 with probabilities 0.2, 0.5, 0.3. For $Y = 6 - X$, find $P(Y \geq 4)$. [2 marks]

$6 - X \geq 4$ gives $X \leq 2$.

Add probabilities at $X = 1$ and $X = 2$: $0.2 + 0.5$.

Answer: 0.7.