Variance and standard deviation

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. X is 0 or 2 with equal probabilities. Find its mean and variance. [2 marks]

$E(X) = 1;\ E(X^2) = 2.$

$\text{Var}(X) = 2 - 1^2 = 1.$

Answer: Mean 1; variance 1.

2. $E(X) = 3$ and $E(X^2) = 14$. Find variance and standard deviation. [2 marks]

$\text{Var}(X) = 14 - 9 = 5.$

Standard deviation is the non-negative square root.

Answer: $5;\ \sqrt{5} \approx 2.2361.$

3. X is -1 or 1 with equal probabilities. Find its mean and standard deviation. [2 marks]

$E(X) = \frac{-1+1}{2} = 0.$

X^2 is always 1, so variance is 1 and standard deviation is 1.

Answer: Mean 0; standard deviation 1.

4. A payout in dollars has variance 9. State its standard deviation and units. [2 marks]

Standard deviation $= \sqrt{9} = 3$.

Its units are dollars, unlike the squared-dollar variance.

Answer: 3 dollars.

5. X has values 1, 2 and 3 with probabilities $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$. Find $E(X^2)$ and $\mathrm{Var}(X)$. [3 marks]

$E(X) = \frac{1}{4} + 1 + \frac{3}{4} = 2$.

$E(X^2) = \frac{1}{4} + 2 + \frac{9}{4} = \frac{9}{2}$.

$\mathrm{Var}(X) = \frac{9}{2} - 4 = \frac{1}{2}$.

Answer: $E(X^2) = \frac{9}{2};\ \mathrm{Var}(X) = \frac{1}{2}$.

6. A student obtains $\mathrm{Var}(X) = -0.8$ from a valid exact probability table. Explain why this cannot be accepted. [2 marks]

Every squared deviation is non-negative and probabilities are non-negative.

Their weighted sum cannot be below zero; recheck the calculation.

Answer: Variance cannot be negative.