Expected value

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. X takes values 1, 2 and 4 with probabilities 0.2, 0.5 and 0.3. Find $E(X)$. [2 marks]

$E(X) = 1(0.2) + 2(0.5) + 4(0.3)$.

The sum is $0.2 + 1 + 1.2 = 2.4$.

Answer: 2.4.

2. A fair die is rolled and X is its score. Find $E(X)$. [2 marks]

Each score has probability $\frac{1}{6}$.

$E(X) = \frac{1+2+3+4+5+6}{6} = \frac{21}{6}$.

Answer: 3.5.

3. A ticket pays 20 dollars with probability 0.1 and 0 dollars otherwise. It costs 3 dollars. Find expected net gain. [2 marks]

Expected prize is $20(0.1) = 2$ dollars.

Net expected gain $= 2 - 3 = -1$ dollar.

Answer: -1 dollar.

4. X is 0 or 5. If $E(X) = 2$, find $P(X = 5)$. [2 marks]

Let $p = P(X = 5)$, so $E(X) = 0(1 - p) + 5p$.

Solve $5p = 2$ to obtain $p = 0.4$.

Answer: 0.4.

5. A machine makes 0, 1 or 2 faulty items per batch with probabilities 0.7, 0.2 and 0.1. Find the expected number over 50 batches. [3 marks]

Expected faults per batch $= 0.2 + 2(0.1) = 0.4$.

By linearity of expectation, the expected total is $50(0.4) = 20$.

Answer: 20 faulty items.

6. A forecast says $E(X) = 2.4$ calls per minute. Explain why 2.4 need not be a possible call count. [2 marks]

Each observed count is an integer.

A probability-weighted mean can be non-integer without contradicting the model.

Answer: It is a long-run average, not an individual count.