Integration - Applications and reasoning test

Name: _____ Date: _____

10 questions | 49 marks | 75 minutes

A harder original applications paper testing choice of method, model assumptions and mathematical explanation.

A scientific calculator may be used.

1. A parameter appears in an integral. [4 marks]

(a) If $\int_0^2 (kx+1)\,dx = 8$, find k. [2 marks]

(b) Find the average value of $kx+1$ over $[0, 2]$ for this k. [2 marks]

2. A student proposes $F(x) = \ln(2x+1)$ as an antiderivative of $\frac{1}{2x+1}$ on $x > 0$. [4 marks]

(a) Identify and explain the error. [2 marks]

(b) Correct the antiderivative and evaluate the integral from 0 to 1. [2 marks]

3. A quantity satisfies $N'(t) = 4e^{2t}$ and $N(0) = 7$. [5 marks]

(a) Find $N(t)$. [2 marks]

(b) Find when $N = 13$. [3 marks]

4. A rate is recorded at 0,1 and 2 hours. [6 marks]

Time (h)	0	1	2
Rate (units/h)	2	5	10

(a) Estimate the accumulated change by trapeziums. [2 marks]

(b) Can the true change be determined exactly from these values alone? Explain. [2 marks]

(c) Name useful additional information. [2 marks]

5. $A(x) = \int_0^{x^2} (t+1)\,dt$. [4 marks]

(a) Find $A'(x)$. [2 marks]

(b) Find all stationary inputs and explain why there are no others. [2 marks]

6. The graphs $y = x$ and $y = -x$ are considered on $[-2, 1]$. [5 marks]

(a) Find where the upper boundary changes. [2 marks]

(b) Find the geometric area. [3 marks]

7. The graph $y = x^2 - 1$ lies below the axis on $[-1, 1]$. [6 marks]

(a) Find its geometric area on this interval. [2 marks]

(b) Find $b > 1$ such that its signed integral from -1 to b is 0. [4 marks]

8. A tank starts with 15 L and has net rate $6 - 2t$ L/min for $0 \le t \le 5$. [5 marks]

(a) Find its maximum volume. [2 marks]

(b) Find its final volume. Explain why the total inflow cannot be determined from this net-rate model. [3 marks]

9. The temperature model is $T(t) = t^2 + 2$°C over $[0, 3]$. [4 marks]

(a) Find the time-average temperature. [2 marks]

(b) Compare with the average of endpoint temperatures. [2 marks]

10. A particle has $v(t) = t^2 - 4t + 3$ m/s on $[0, 4]$, initial position 2 m. [6 marks]

(a) Find its final position. [2 marks]

(b) Find the total distance and explain why it is larger than the magnitude of displacement. [4 marks]