Integration - Practice test 5

Name: _____ Date: _____

10 questions | 45 marks | 70 minutes

A modelling paper connecting rates, marginal values, averages and particle movement.

A scientific calculator may be used.

1. Find and verify. [4 marks]

(a) Find $\int 1\,dx$. [2 marks]

(b) Find $\int (2x+3)\,dx$. [2 marks]

2. $f(x)=\frac{4}{2x+1}$ on $x\geq 0$. [4 marks]

(a) Find an antiderivative. [2 marks]

(b) Find its integral from 0 to 2. [2 marks]

3. A cumulative quantity has derivative $Q'(t) = 6t^2$ and $Q(1) = 10$. [4 marks]

(a) Find $Q(t)$. [2 marks]

(b) Find the change from 1 to 2. [2 marks]

4. Three rate samples are supplied. [4 marks]

Time (min)	0	2	4
Flow (L/min)	3	7	13

(a) Use two trapeziums to estimate total flow over 4 min. [2 marks]

(b) Would equal-width left rectangles give a lower estimate than the trapezium estimate in part (a)? Calculate it and explain. [2 marks]

5. $F(x) = \int_1^{3x} t^2\,dt$. [4 marks]

(a) Find $F'(x)$. [2 marks]

(b) Evaluate $F(\frac{1}{3})$. [2 marks]

6. The line $y = x$ is considered between $x = -2$ and $x = 3$. [4 marks]

(a) Find its signed integral. [2 marks]

(b) Find its geometric area. [2 marks]

7. The curves $y = 3x$ and $y = x^2$ enclose a region. [5 marks]

(a) Find the intersection points. [2 marks]

(b) Calculate area. [3 marks]

8. A workshop has marginal cost $C'(q) = 2q + 5$ and fixed cost 40 dollars. [6 marks]

(a) Find total cost at 6 items. [2 marks]

(b) Find the average cost per item at 6 items. [2 marks]

(c) Distinguish average cost from marginal cost at 6 items. [2 marks]

9. For $f(t) = 3t^2$ on $[0, 2]$. [4 marks]

(a) Find its average value. [2 marks]

(b) Find a time when the function equals that average. [2 marks]

10. A particle has acceleration $a(t) = 2$ m/s², initial velocity -2 m/s and initial position 5 m, on $[0, 3]$. [6 marks]

(a) Find its velocity and direction-change time. [2 marks]

(b) Find final position and distance. [4 marks]