Integration - Practice test 5

10 questions | 45 marks | 70 minutes

A modelling paper connecting rates, marginal values, averages and particle movement.

Equivalent correct methods earn the corresponding marks.

1. Find and verify. [4 marks]

(a) Find $\int 1\,dx$. [2 marks]

The constant integrand 1 has antiderivative $x + C$.

Answer: $x + C$

(b) Find $\int (2x + 3)\,dx$. [2 marks]

Integrate the expanded linear expression.

Answer: $x^2 + 3x + C$

2. $f(x) = \frac{4}{2x+1}$ on $x \geq 0$. [4 marks]

(a) Find an antiderivative. [2 marks]

Divide 4 by 2.

Answer: $2\ln(2x + 1) + C$

(b) Find its integral from 0 to 2. [2 marks]

Evaluate upper minus lower.

Answer: $2\ln 5$

3. A cumulative quantity has derivative $Q'(t) = 6t^2$ and $Q(1) = 10$. [4 marks]

(a) Find $Q(t)$. [2 marks]

$10 = 2 + C$.

Answer: $2t^3 + 8$

(b) Find the change from 1 to 2. [2 marks]

$Q(2) - Q(1) = 24 - 10 = 14$.

Answer: 14

4. Three rate samples are supplied. [4 marks]

Time (min)	0	2	4
Flow (L/min)	3	7	13

(a) Use two trapeziums to estimate total flow over 4 min. [2 marks]

Width 2 gives $(3 + 7) + (7 + 13) = 30$ L.

Answer: 30 L

(b) Would equal-width left rectangles give a lower estimate than the trapezium estimate in part (a)? Calculate it and explain. [2 marks]

The width is 2 min, so $L = 2(3 + 7) = 20$ L. [1 mark]

This is 10 L below the trapezium estimate of 30 L. Without between-sample information, it is not a proven lower bound on the true accumulated flow. [1 mark]

Answer: Yes: $L = 20$ L, which is below the trapezium estimate of 30 L.

5. $F(x) = \int_1^{3x} t^2\,dt$. [4 marks]

(a) Find $F'(x)$. [2 marks]

$(3x)^2 \cdot 3 = 27x^2$.

Answer: $27x^2$

(b) Evaluate $F(\frac{1}{3})$. [2 marks]

Both limits are 1.

Answer: 0

6. The line $y = x$ is considered between $x = -2$ and $x = 3$. [4 marks]

(a) Find its signed integral. [2 marks]

$[\frac{x^2}{2}]_{-2}^{3} = \frac{9}{2} - 2$.

Answer: $\frac{5}{2}$

(b) Find its geometric area. [2 marks]

Separate the triangles of area 2 and $\frac{9}{2}$.

Answer: $\frac{13}{2}$

7. The curves $y = 3x$ and $y = x^2$ enclose a region. [5 marks]

(a) Find the intersection points. [2 marks]

$x(x-3) = 0$.

Answer: $(0, 0)$ and $(3, 9)$

(b) Calculate area. [3 marks]

$\int_0^3 (3x - x^2)\,dx = \frac{9}{2}$.

Answer: $\frac{9}{2}$

8. A workshop has marginal cost $C'(q) = 2q + 5$ and fixed cost 40 dollars. [6 marks]

(a) Find total cost at 6 items. [2 marks]

Additional cost $\int_0^6 (2q + 5)\,dq = 66$.

Answer: 106 dollars

(b) Find the average cost per item at 6 items. [2 marks]

Divide total 106 by 6.

Answer: $\frac{53}{3}$ dollars/item

(c) Distinguish average cost from marginal cost at 6 items. [2 marks]

Average uses total cost divided by output; marginal is the model rate $C'(6)$.

Answer: Average $\frac{53}{3}$; marginal 17 dollars/item.

9. For $f(t) = 3t^2$ on $[0, 2]$. [4 marks]

(a) Find its average value. [2 marks]

Integral 8 divided by interval length 2.

Answer: 4

(b) Find a time when the function equals that average. [2 marks]

Solve $3t^2 = 4$ with $t \geq 0$.

Answer: $t = \frac{2}{\sqrt{3}}$

10. A particle has acceleration $a(t) = 2$ m/s², initial velocity -2 m/s and initial position 5 m, on $[0, 3]$. [6 marks]

(a) Find its velocity and direction-change time. [2 marks]

Integrate acceleration, use $v(0) = -2$, then solve $v = 0$.

Answer: $v = 2t - 2$; reversal at 1 s.

(b) Find final position and distance. [4 marks]

Displacement $[t^2 - 2t]_0^3 = 3$ m; add 5 m.

Signed pieces -1 and 4 give total distance 5 m.

Answer: 8 m position;5 m distance.