Integration - Practice test 4

Name: _____ Date: _____

10 questions | 44 marks | 70 minutes

A varied fourth paper emphasising trigonometric, exponential and logarithmic accumulation.

A scientific calculator may be used.

1. Reverse differentiation. [4 marks]

(a) Find $\int(4x^3 - 2e^{-x})\,dx$. [2 marks]

(b) Find $\int 5\cos(5x)\,dx$. [2 marks]

2. Logarithmic forms. [4 marks]

(a) Find $\int \frac{3}{2-x}\,dx$. [2 marks]

(b) Evaluate $\int_0^1 \frac{3}{2-x}\,dx$. [2 marks]

3. A curve satisfies $y' = 2\sin x$ and $y(0) = 3$. [4 marks]

(a) Find y. [2 marks]

(b) Find its net change from 0 to π. [2 marks]

4. A continuous function has $\int_0^3 f = 4$. [4 marks]

(a) Find $\int_0^3 (2f(x) + 1)\,dx$. [2 marks]

(b) Find $\int_3^0 (2f(x) + 1)\,dx$. [2 marks]

5. $A(x) = \int_0^{x+2} e^t \, dt$. [4 marks]

(a) Find A'. [2 marks]

(b) Evaluate $A(0)$. [2 marks]

6. For $f(x) = x^2$ on $[0, 2]$. [6 marks]

(a) Calculate the exact integral. [2 marks]

(b) The two-strip trapezium estimate is 3. Find the absolute error. [2 marks]

(c) Explain the direction of this error. [2 marks]

7. Consider $y = \sin x$ on $[0, 2\pi]$. [5 marks]

(a) Find the signed integral. [2 marks]

(b) Find geometric area. [3 marks]

8. Find the bounded region between $y = x$ and $y = x^2$. [5 marks]

(a) Identify which curve is higher between intersections. [2 marks]

(b) Find area. [3 marks]

9. A tank receives $12e^{-t}$ L/min for $0 \leq t \leq \ln 2$. [4 marks]

(a) Find the supplied volume. [2 marks]

(b) Find the average flow rate. [2 marks]

10. A particle has $v(t) = \cos t$ m/s on $[0, \pi]$ and initial position 2 m. [4 marks]

(a) Find its final position. [2 marks]

(b) Find its distance. [2 marks]