Integration - Practice test 3

Name: _____ Date: _____

10 questions | 47 marks | 70 minutes

Reasoning, sign changes and model interpretation, with several linked calculations.

A scientific calculator may be used.

1. Choose the rule. [4 marks]

(a) Find $\int(3\sqrt{x}+\frac{2}{x})\,dx$ for $x>0$. [2 marks]

(b) State why the same power rule cannot be used for both terms. [2 marks]

2. $F(x)=2\ln(3x+1)$ for $x>-\frac{1}{3}$. [4 marks]

(a) Find $F'(x)$. [2 marks]

(b) Hence evaluate $\int_0^1 \frac{6}{3x+1}\,dx$. [2 marks]

3. $f'(x) = 4x - 1$, with $f(2) = 5$. [4 marks]

(a) Find f. [2 marks]

(b) Verify both pieces of given information. [2 marks]

4. A signed integral has been split into intervals. [6 marks]

(a) If $\int_0^1 f = -2$ and $\int_1^4 f = 7$, find $\int_0^4 f$. [2 marks]

(b) If $f \leq 0$ on $[0, 1]$ and $f \geq 0$ on $[1, 4]$, find its geometric area. [2 marks]

(c) Without those sign conditions, is 9 necessarily the total area? Explain. [2 marks]

5. $A(x) = \int_2^{1-x} t^2\,dt$. [4 marks]

(a) Find $A'(x)$. [2 marks]

(b) Find $A(-1)$. [2 marks]

6. A pump rate is sampled in the supplied table. [4 marks]

Time (min)	0	2	4	6
Rate (L/min)	4	8	12	10

(a) Estimate total volume from 0 to 6 min by trapeziums. [2 marks]

(b) Find the average estimated flow rate. [2 marks]

7. The graph $y = x(x-2)$ is considered on $[0, 3]$. [5 marks]

(a) Find the signed integral. [2 marks]

(b) Find its geometric area. [3 marks]

8. The curves $y = 4 - x^2$ and $y = x + 2$ enclose a region. [5 marks]

(a) Show that the bounds are -2 and 1. [2 marks]

(b) Find the area. [3 marks]

9. A stock changes at $r(t) = 6 - 2t$ units/day over 4 days and starts at 100 units. [5 marks]

(a) Find when the stock is largest. [2 marks]

(b) Find the largest stock and the stock at day 4. [3 marks]

10. A particle has $v(t) = (t-1)(t-3)$ m/s over $[0, 4]$. [6 marks]

(a) Identify direction-change times. [2 marks]

(b) Find displacement and distance. [4 marks]