Integration - Practice test 3

10 questions | 47 marks | 70 minutes

Reasoning, sign changes and model interpretation, with several linked calculations.

Equivalent correct methods earn the corresponding marks.

1. Choose the rule. [4 marks]

(a) Find $\int(3\sqrt{x}+\frac{2}{x})\,dx$ for $x>0$. [2 marks]

Use different rules for the two terms.

Answer: $2x^{\frac{3}{2}}+2\ln x+C$

(b) State why the same power rule cannot be used for both terms. [2 marks]

The power-rule denominator would be 0.

Answer: The exponent -1 requires the logarithmic rule.

2. $F(x)=2\ln(3x+1)$ for $x>-\frac{1}{3}$. [4 marks]

(a) Find $F'(x)$. [2 marks]

Apply the log chain rule.

Answer: $\frac{6}{3x+1}$

(b) Hence evaluate $\int_0^1 \frac{6}{3x+1}\,dx$. [2 marks]

$F(1)-F(0)=2\ln 4.$

Answer: $2\ln 4$

3. $f'(x) = 4x - 1$, with $f(2) = 5$. [4 marks]

(a) Find f. [2 marks]

$5 = 8 - 2 + C$ gives $C = -1$.

Answer: $2x^2 - x - 1$

(b) Verify both pieces of given information. [2 marks]

Differentiate and substitute independently.

Answer: Derivative $4x - 1$ and $f(2) = 5$.

4. A signed integral has been split into intervals. [6 marks]

(a) If $\int_0^1 f = -2$ and $\int_1^4 f = 7$, find $\int_0^4 f$. [2 marks]

Add the signed quantities.

Answer: 5

(b) If $f \leq 0$ on $[0, 1]$ and $f \geq 0$ on $[1, 4]$, find its geometric area. [2 marks]

Add magnitudes $2 + 7$.

Answer: 9

(c) Without those sign conditions, is 9 necessarily the total area? Explain. [2 marks]

Further sign changes could make individual interval integrals conceal additional area.

Answer: No.

5. $A(x)=\int_2^{1-x} t^2\,dt$. [4 marks]

(a) Find $A'(x)$. [2 marks]

Endpoint derivative -1.

Answer: $-(1-x)^2$

(b) Find $A(-1)$. [2 marks]

Upper and lower limits are both 2.

Answer: 0

6. A pump rate is sampled in the supplied table. [4 marks]

Time (min)	0	2	4	6
Rate (L/min)	4	8	12	10

(a) Estimate total volume from 0 to 6 min by trapeziums. [2 marks]

Width 2: $(4+8)+(8+12)+(12+10)=54$.

Answer: 54 L

(b) Find the average estimated flow rate. [2 marks]

Divide 54 by 6.

Answer: 9 L/min

7. The graph $y=x(x-2)$ is considered on $[0,3]$. [5 marks]

(a) Find the signed integral. [2 marks]

$[\frac{x^3}{3}-x^2]_0^3=0$.

Answer: 0

(b) Find its geometric area. [3 marks]

Split at 2; contributions $-\frac{4}{3}$ and $\frac{4}{3}$.

Answer: $\frac{8}{3}$

8. The curves $y = 4 - x^2$ and $y = x + 2$ enclose a region. [5 marks]

(a) Show that the bounds are -2 and 1. [2 marks]

Equate the functions.

Answer: $x^2 + x - 2 = (x + 2)(x - 1) = 0$

(b) Find the area. [3 marks]

Integrate $2 - x - x^2$ on $[-2, 1]$.

Answer: $\frac{9}{2}$

9. A stock changes at $r(t) = 6 - 2t$ units/day over 4 days and starts at 100 units. [5 marks]

(a) Find when the stock is largest. [2 marks]

The rate changes from positive to negative at 3.

Answer: Day 3

(b) Find the largest stock and the stock at day 4. [3 marks]

Change to 3 is $[6t - t^2]_0^3 = 9$; to 4 is 8.

Answer: 109 units and 108 units.

10. A particle has $v(t) = (t - 1)(t - 3)$ m/s over $[0, 4]$. [6 marks]

(a) Identify direction-change times. [2 marks]

Both simple zeros change the sign.

Answer: 1 s and 3 s

(b) Find displacement and distance. [4 marks]

Signed pieces $\frac{4}{3}, -\frac{4}{3}, \frac{4}{3}$.

Answer: $\frac{4}{3}$ m and 4 m