Integration - Practice test 2

10 questions | 50 marks | 70 minutes

A second balanced paper with flow, cost and graphical reasoning.

Equivalent correct methods earn the corresponding marks.

1. Powers and constants. [4 marks]

(a) Find $\int(6x^2+4x-3)\,dx$. [2 marks]

Use the power rule term by term.

Answer: $2x^3+2x^2-3x+C$

(b) Explain why different constants give the same derivative. [2 marks]

The derivative therefore does not fix vertical position.

Answer: The derivative of a constant is 0.

2. Exponential and trigonometric forms. [4 marks]

(a) Find $\int 5e^{-x}\,dx$. [2 marks]

Divide by input coefficient -1.

Answer: $-5e^{-x}+C$

(b) Find $\int 6\sin(3x)\,dx$. [2 marks]

Use negative cosine and divide by 3.

Answer: $-2\cos(3x)+C$

3. $g'(x)=\frac{2}{x}$ on $x>0$, with $g(e)=5$. [4 marks]

(a) Find g. [2 marks]

$5=2+C$ gives 3.

Answer: $g=2\ln x+3$

(b) Find $g(1)$. [2 marks]

$\ln 1=0$.

Answer: 3

4. A tank starts with 60 L. Its inflow is $9+t$ L/min and outflow 4 L/min for 0 to 4 min. [6 marks]

(a) Write and evaluate the integral for net change. [2 marks]

$\int_0^4 (5+t)\,dt = 28.$

Answer: 28 L

(b) Find the final volume. [2 marks]

Add 60 L.

Answer: 88 L

(c) Find the total inflow. [2 marks]

$\int_0^4 (9+t)\,dt = 36+8 = 44.$

Answer: 44 L

5. For $f(x)=x^2$ on $[0,2]$, use two equal strips. [6 marks]

(a) Find the left-rectangle estimate. [2 marks]

Heights 0 and 1; width 1.

Answer: 1

(b) Find the right-rectangle estimate. [2 marks]

Heights 1 and 4.

Answer: 5

(c) Explain why the exact integral lies between them. [2 marks]

Each left rectangle is below and each right rectangle above the curve.

Answer: f is increasing and nonnegative.

6. $B(x) = \int_1^{x^2} (t+2)\,dt$. [4 marks]

(a) Find $B'(x)$. [2 marks]

Apply endpoint substitution and the chain rule.

Answer: $2x(x^2+2)$

(b) Evaluate $B(1)$. [2 marks]

Equal bounds give 0.

Answer: 0

7. The curve $y = 2 - x$ is considered on $[0, 5]$. [6 marks]

(a) Find its zero. [2 marks]

$2 - x = 0$.

Answer: $x = 2$

(b) Find signed integral and geometric area. [4 marks]

$[2x - \frac{x^2}{2}]_0^5 = -\frac{5}{2}$.

Geometric pieces 2 and $\frac{9}{2}$ sum to $\frac{13}{2}$.

Answer: $-\frac{5}{2}$ and $\frac{13}{2}$

8. Find the region between $y = 4$ and $y = x^2$. [5 marks]

(a) Find the bounds. [2 marks]

$x^2 = 4$.

Answer: $x = -2, 2$

(b) Calculate its area. [3 marks]

$\int_{-2}^{2} (4 - x^2)\,dx = \frac{32}{3}$.

Answer: $\frac{32}{3}$

9. A production-cost model has $C'(q) = 0.06q + 4$ dollars/item and $C(0) = 80$ dollars. [6 marks]

(a) Find $C(q)$. [2 marks]

Integrate and use the fixed cost.

Answer: $0.03q^2 + 4q + 80$

(b) Find the cost of producing 10 items. [2 marks]

$3 + 40 + 80 = 123.$

Answer: 123

(c) Explain what $\int_{10}^{20} C'(q)\,dq$ measures. [2 marks]

It excludes the already incurred cost at 10 items.

Answer: Additional cost when output rises from 10 to 20 items.

10. A particle has $v(t) = 2t - 4$ m/s on $[0, 4]$. [5 marks]

(a) Find its displacement. [2 marks]

$[t^2 - 4t]_0^4 = 0.$

Answer: 0 m

(b) Find its distance. [3 marks]

Split at 2; two triangular areas each 4 m.

Answer: 8 m