Areas between curves

6 questions | 18 marks

Equivalent correct methods earn the corresponding marks.

1. Find the bounded area between $y = x$ and $y = x^2$. [3 marks]

Bounds 0 and 1; upper minus lower $x - x^2$.

$[\frac{x^2}{2} - \frac{x^3}{3}]_0^1 = \frac{1}{6}$.

Answer: $\frac{1}{6}$

2. Find the bounded area between $y = 4$ and $y = x^2$. [3 marks]

Intersections at ± 2.

Integrate $4 - x^2$ on $[-2, 2]$.

Answer: $\frac{32}{3}$

3. Find the area between $y = 3x$ and $y = x$ on $[1, 3]$. [3 marks]

Integrate $2x$ from 1 to 3: $[x^2]_1^3 = 8$.

Answer: 8

4. Find the bounded area between $y = 3x$ and $y = x^2$. [3 marks]

Bounds 0 and 3.

$\int_0^3 (3x - x^2)\,dx = \frac{27}{2} - 9 = \frac{9}{2}$.

Answer: $\frac{9}{2}$

5. Find the area between $y = x$ and $y = -x$ on $[-1, 1]$. [3 marks]

The upper curve changes at 0.

$\int_{-1}^0 (-2x)\,dx + \int_0^1 2x\,dx = 1 + 1 = 2$.

Answer: 2

6. Explain why $\int_0^2 (x^2 - 2x)\,dx$ is not the geometric area between $y = x^2$ and $y = 2x$. [3 marks]

It gives $-\frac{4}{3}$; the geometric area is $\frac{4}{3}$.

Answer: The integrand is lower minus upper.