Numerical area approximation and the integral idea

6 questions | 18 marks

Equivalent correct methods earn the corresponding marks.

1. For $y = x^2$ on $[0, 2]$, use two right rectangles. [3 marks]

Width 1; heights 1 and 4 give 5.

Answer: 5

2. For $y = x^2$ on $[0, 2]$, use midpoint rectangles of width 1. [3 marks]

Heights $f(0.5) = 0.25$ and $f(1.5) = 2.25$ sum to 2.5.

Answer: $\frac{5}{2}$

3. For $y = x^2$ on $[0, 2]$, use four trapeziums. [3 marks]

$h = 0.5$; heights 0,0.25,1,2.25,4.

$T = 0.25[0 + 2(0.25 + 1 + 2.25) + 4] = 2.75.$

Answer: $\frac{11}{4}$

4. A rate is 4,6 and 10 L/min at 0,2 and 4 min. Estimate total volume by trapeziums. [3 marks]

$T = \frac{2(4+6)}{2} + \frac{2(6+10)}{2} = 10 + 16 = 26.$

Answer: 26 L

5. The function $f(x) = 2x + 1$ is sampled at 0,1 and 2. Is the trapezium estimate exact? Explain. [3 marks]

A straight line matches each trapezium top exactly.

$T = \frac{1+3}{2} + \frac{3+5}{2} = 6.$

Answer: Yes; its value is 6.

6. Why does the trapezium rule overestimate $\int_0^2 x^2\,dx$? [3 marks]

Each chord between neighbouring endpoints lies above the curve, so the trapezium includes extra area.

Answer: The graph is convex.