Antiderivatives involving reciprocal linear expressions, linked back to logarithmic derivatives

6 questions | 18 marks

Equivalent correct methods earn the corresponding marks.

1. Find $\int \frac{5}{x}\,dx$. [3 marks]

Use $\int \frac{1}{x}\,dx = \ln|x| + C$.

Answer: $5\ln|x| + C$

2. Find $\int \frac{2}{4x+1}\,dx$. [3 marks]

Divide 2 by the input coefficient 4.

Answer: $\frac{1}{2}\ln|4x+1| + C$

3. Find $\int \frac{1}{5-2x}\,dx$. [3 marks]

The denominator derivative is -2; divide by -2.

Answer: $-\frac{1}{2}\ln|5-2x| + C$

4. Find $\int (x + \frac{2}{x})\,dx$ on $x > 0$. [3 marks]

Use the power rule for x and log rule for $\frac{1}{x}$.

Answer: $\frac{x^2}{2} + 2\ln x + C$

5. Find $\int \frac{x+3}{x}\,dx$ on $x < 0$. [3 marks]

Simplify to $1 + \frac{3}{x}$.

The absolute value gives a real logarithm on $x < 0$.

Answer: $x + 3\ln|x| + C$

6. Can $\int_{-1}^{1} \frac{1}{x}\,dx$ be evaluated by substituting into $\ln|x|$ under the ordinary definite-integral rules? Explain. [3 marks]

The integrand is undefined at 0 inside the interval.

The continuity condition for the usual fundamental-theorem evaluation fails.

Answer: No.