Logarithmic functions - Practice test 6

Name: _____ Date: _____

10 questions | 61 marks | 70 minutes

A more demanding paper combining precise domains, parameter choices and mathematical reasoning.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. Solve and justify the base restrictions. [5 marks]

(a) Find b if $\log_b\left(\frac{1}{16}\right) = 4$. [2 marks]

(b) Explain why base 1 cannot define a logarithm on all positive inputs. [3 marks]

2. Consider $F(x) = \ln(x-1) + \ln(x+3)$. [6 marks]

(a) State its domain and solve $F(x) = \ln(12)$. [4 marks]

(b) Explain why solving the combined equation may introduce an inadmissible branch. [2 marks]

3. Solve $e^{x} + 3e^{-x} = 4$. [5 marks]

(a) Find all possible positive values of $u = e^{x}$. [3 marks]

(b) Find all x and decide which is larger. [2 marks]

4. A family is $f_a(x) = \ln(x - a) + 2a$. [6 marks]

(a) Find the domain and x-intercept in terms of a. [3 marks]

(b) Find the inverse function and state its range. [3 marks]

5. Let $g(x) = \ln x - kx$ for $x > 0$ and $k > 0$. [7 marks]

(a) Find the stationary point in terms of k and classify it. [4 marks]

(b) Find k so that the maximum value is zero. [3 marks]

6. Consider $h(x) = \ln\left(\frac{x+1}{x-1}\right)$. [6 marks]

(a) Find its real domain. [3 marks]

(b) Differentiate h on its entire domain without splitting it into invalid logarithms. [3 marks]

7. The curve is $y = \ln(2 + x^2)$. [7 marks]

(a) Find the tangent at $x = 1$. [3 marks]

(b) Find the two points at which the tangent gradient equals $\frac{1}{2}$. [4 marks]

8. A response is $R(t) = A + B\ln(1 + \frac{t}{c})$ with $B, c > 0$ and $t \geq 0$. Time is in hours. [7 marks]

(a) Show that its derivative is $\frac{B}{c+t}$ and give the rate units. [3 marks]

(b) If the initial rate is 6 and the rate at $t = 4$ is 2, find B and c. [4 marks]

9. A model score is $S(x) = 10\log_{10}\left(\frac{x}{x_0}\right)$ for positive x and x_0. [6 marks]

(a) A change of 7 scale units occurs. Find the exact multiplier of x. [3 marks]

(b) Show that adding equal amounts to x does not generally give equal changes in S. [3 marks]

10. A proposed simplification and an asymptote need careful interpretation. [6 marks]

(a) Compare the functions $p(x) = \ln(x^2 - 1)$ and $q(x) = \ln(x - 1) + \ln(x + 1)$. Where do they agree and where is only p defined? [3 marks]

(b) For p, find its derivative and the limiting behaviour as $x \to -1^-$ and $x \to 1^+$. [3 marks]