Logarithmic functions - Practice test 5

10 questions | 57 marks | 70 minutes

Data interpretation, parameter fitting and checking claims about logarithmic models.

Equivalent correct methods earn the corresponding marks.

1. Use a supplied table of powers. [5 marks]

Selected exact powers

n	3^n
-2	$\frac{1}{9}$
0	1
1	3
2	9
3	27

(a) Fill the logarithmic values for inputs $\frac{1}{9}$, 1 and 27 in base 3. [2 marks]

The exponents of 3 are $-2, 0, 3$.

Answer: $-2, 0, 3$.

(b) Estimate whether $\log_3 12$ lies between 1 and 2 or between 2 and 3, then calculate it to three decimal places. [3 marks]

$9 < 12 < 27$ and base-3 logarithms increase. $\frac{\ln 12}{\ln 3} \approx 2.261860$.

Answer: Between 2 and 3; 2.262.

2. Compare expressions and domains. [5 marks]

(a) For $x > 0$, simplify $\ln(8x) - \ln(2x)$. [2 marks]

The quotient of the inputs is 4.

Answer: $\ln 4$.

(b) Is $\ln(x-2) + \ln(x+2) = \ln(x^2-4)$ valid for every x in the domain of the right side? Explain. [3 marks]

An algebraic product identity does not remove individual log restrictions.

Answer: No. The left side requires $x > 2$; the right side also allows $x < -2$.

3. A coefficient is hidden inside an exponential model. [6 marks]

(a) Find k if $40e^{3k} = 100$. [3 marks]

$e^{3k} = 2.5$, then take natural logarithms.

Answer: $k = \frac{\ln(2.5)}{3} \approx 0.3054$.

(b) Using this exact k, solve $40e^{kt} = 250$. [3 marks]

$6.25 = (2.5)^2$, so $kt = 2\ln(2.5)$ and $t = 6$.

Answer: $t = \frac{3\ln(6.25)}{\ln(2.5)} = 6$.

4. The inverse function of f is $f^{-1}(x) = 4 + e^{\frac{x}{2}}$. [5 marks]

(a) Find f and its domain. [3 marks]

Solve $y = 4 + e^{\frac{x}{2}}$ for x, then interchange names.

Answer: $f(x) = 2\ln(x-4),\ x > 4$.

(b) Find the asymptote and the point at which $f(x) = 0$. [2 marks]

The input tends to zero at 4; $\ln(x-4) = 0$ gives input 1.

Answer: Asymptote $x = 4$; point $(5, 0)$.

5. Let $f(x) = \ln(x^2 + 4x + 5)$. [6 marks]

(a) Explain why its domain is all real numbers. [2 marks]

Complete the square.

Answer: $x^2 + 4x + 5 = (x+2)^2 + 1 > 0$.

(b) Find its stationary point and classify it. [4 marks]

$f' = \frac{2x+4}{x^2+4x+5}$. Its sign changes from negative to positive at -2.

Answer: $(-2, 0)$, a minimum.

6. A response is modelled by $R(t) = a + b\ln(t+1)$ for $t \geq 0$ minutes. Observations are supplied. [7 marks]

Response observations

t (minutes)	R
0	5
$e-1$	13

(a) Find a and b exactly. [3 marks]

$R(0) = a = 5$. At $t = e-1$, $\ln(t+1) = 1$, so $b = 8$.

Answer: $a = 5$, $b = 8$.

(b) Find the time when $R = 25$ and the rate at that time. [4 marks]

$8\ln(t+1) = 20$. The derivative is $\frac{8}{t+1}$.

Answer: $t = e^{\frac{5}{2}} - 1$ minutes; rate $8e^{-\frac{5}{2}}$ response units per minute.

7. Two graphs are $y = \ln x$ and $y = \ln x + \ln 5$. [5 marks]

(a) Rewrite the second as one logarithm and describe its horizontal scaling from the first. [3 marks]

Multiplying the input by 5 divides each old x-coordinate by 5.

Answer: $y = \ln(5x)$; horizontal scale factor $\frac{1}{5}$.

(b) Explain how the same graph can also be described by a vertical translation. [2 marks]

The logarithm law makes the two descriptions equivalent.

Answer: It is translated up by $\ln 5$.

8. A supplied scale is $B = \log_2\left(\frac{N}{N_0}\right)$ for positive N and fixed positive N_0. [5 marks]

(a) Find $\frac{N}{N_0}$ when $B = -3$. [2 marks]

Exponentiate base 2.

Answer: $\frac{1}{8}$.

(b) Find the change in B if N is multiplied by 12. [3 marks]

Subtract the original scale value and combine the quotient.

Answer: $\log_2 12 \approx 3.585$.

9. A claimed derivative for $g(x) = \ln(4 - x^2)$ is $\frac{1}{4 - x^2}$. [6 marks]

(a) Give the correct derivative and domain. [3 marks]

Use the chain rule and require a positive input.

Answer: $g' = -\frac{2x}{4 - x^2}$; $-2 < x < 2$.

(b) Use the correct derivative to determine where g is increasing. [3 marks]

The denominator is positive on the domain, so $g' > 0$ exactly when $x < 0$.

Answer: $-2 < x < 0$.

10. A fitted score model is $S(t) = 25 + 15\ln(t + 1)$, based on observations for $0 \le t \le 6$ weeks. [7 marks]

(a) Predict the score at $t = 6$ and its instantaneous rate there. [3 marks]

Substitute 6, then use $S' = \frac{15}{t + 1}$.

Answer: $25 + 15\ln 7 \approx 54.2$; rate $\frac{15}{7} \approx 2.14$ points per week.

(b) A researcher says the score will pass 100 eventually. Find the mathematical threshold and give a reason this may not be a reliable practical claim. [4 marks]

$15\ln(t + 1) > 75$ gives $t > e^5 - 1$. Mathematical growth alone does not validate long-term prediction.

Answer: $t = e^5 - 1 \approx 147.4$ weeks; it extrapolates far outside the fitted data and may ignore a maximum score.