Logarithmic functions - Practice test 4

Name: _____ Date: _____

10 questions | 54 marks | 70 minutes

Reasoning about domains, parameters, inverse graphs and an optimisation model.

A scientific calculator may be used.

1. Find real domains before simplifying. [5 marks]

(a) State the domain of $\ln(x+2)+\ln(4-x)$. [2 marks]

(b) Compare it with the domain of $\ln((x+2)(4-x))$. [3 marks]

2. Determine unknown logarithm bases. [5 marks]

(a) Find b if $\log_b 9=\frac{1}{2}$. [2 marks]

(b) Find c if $\log_c 16=-2$. [3 marks]

3. For $u > 0$, define $H(u) = \ln u + \ln(\frac{1}{u})$. [5 marks]

(a) Show that $H(u)$ is constant and state its value. [2 marks]

(b) Differentiate the original two-term expression to confirm the result. [3 marks]

4. Solve $\ln(x^2 - 3x) = \ln 4$. [6 marks]

(a) Find the original domain. [3 marks]

(b) Find and check every real solution. [3 marks]

5. Consider $2^x + 2^{-x} = \frac{5}{2}$. [6 marks]

(a) Use $u = 2^x$ to find a quadratic equation in u. [3 marks]

(b) Find all real x. [3 marks]

6. Let $f(x) = \ln(3x),\ x > 0$. [5 marks]

(a) Find its inverse and describe the relationship between the two graphs. [3 marks]

(b) Find the point on the inverse corresponding to $(\frac{1}{3}, 0)$ on f. [2 marks]

7. Let $f(x) = a\ln x + bx$ for $x > 0$. Its tangent at $x = 1$ is horizontal and $f(1) = 6$. [6 marks]

(a) Determine a and b. [3 marks]

(b) Classify the stationary point at $x = 1$. [3 marks]

8. A mathematical cost model is $C(x) = 20 + x - 4\ln x$ for $x > 0$. [6 marks]

(a) Find the value of x minimising C and justify that it is a global minimum. [3 marks]

(b) Give the minimum exactly and explain why $x = 0$ cannot be tested as an endpoint. [3 marks]

9. Distance travelled in a supplied model is $D(t) = 12\ln(t+1)$ metres, $t \geq 0$ seconds. [5 marks]

(a) Find the time when $D = 12$ metres. [2 marks]

(b) If $t+1$ doubles, show that the additional distance is constant and find it. [3 marks]

10. A student fits a formula without checking its useful domain. [5 marks]

(a) Find the mathematical domain of $M(t) = 50 + 10\ln(t-2)$. [2 marks]

(b) The data were observed only for $3 \leq t \leq 8$ days. Explain two separate cautions about predicting at $t = 1$ and $t = 100$. [3 marks]