# Logarithmic functions - Practice test 4

10 questions | 54 marks | 70 minutes

Reasoning about domains, parameters, inverse graphs and an optimisation model.

Equivalent correct methods earn the corresponding marks.

**1.** Find real domains before simplifying. [5 marks]

**(a)** State the domain of $\ln(x+2)+\ln(4-x)$. [2 marks]

Both logarithm inputs must be positive.

**Answer:** $-2<x<4$.

**(b)** Compare it with the domain of $\ln((x+2)(4-x))$. [3 marks]

The product is positive between its zeros $-2$ and $4$ and negative outside.

**Answer:** The same domain, $-2<x<4$, in this particular case.

**2.** Determine unknown logarithm bases. [5 marks]

**(a)** Find $b$ if $\log_b 9=\frac{1}{2}$. [2 marks]

$b^{\frac{1}{2}}=9$, so square both sides.

**Answer:** $b=81$

**(b)** Find $c$ if $\log_c 16=-2$. [3 marks]

$c^{-2}=16$, giving positive $c=\frac{1}{4}$.

**Answer:** $c=\frac{1}{4}$

**3.** For $u > 0$, define $H(u) = \ln u + \ln(\frac{1}{u})$. [5 marks]

**(a)** Show that $H(u)$ is constant and state its value. [2 marks]

The product of the inputs is $1$, so $H = \ln 1 = 0$.

**Answer:** $H(u) = 0$.

**(b)** Differentiate the original two-term expression to confirm the result. [3 marks]

For the second term, divide $-\frac{1}{u^2}$ by $\frac{1}{u}$.

**Answer:** $H'(u) = \frac{1}{u} - \frac{1}{u} = 0$.

**4.** Solve $\ln(x^2 - 3x) = \ln 4$. [6 marks]

**(a)** Find the original domain. [3 marks]

$x(x - 3) > 0$.

**Answer:** $x < 0$ or $x > 3$.

**(b)** Find and check every real solution. [3 marks]

$x^2 - 3x - 4 = (x - 4)(x + 1) = 0$; both candidates make the input equal to $4$.

**Answer:** $x = -1$ or $4$.

**5.** Consider $2^x + 2^{-x} = \frac{5}{2}$. [6 marks]

**(a)** Use $u = 2^x$ to find a quadratic equation in $u$. [3 marks]

$u + \frac{1}{u} = \frac{5}{2}$; multiply by $2u$.

**Answer:** $2u^2 - 5u + 2 = 0$, with $u > 0$.

**(b)** Find all real $x$. [3 marks]

$(2u - 1)(u - 2) = 0$ gives $u = \frac{1}{2}, 2$.

**Answer:** $x = 1$ or $-1$.

**6.** Let $f(x) = \ln(3x)$, $x > 0$. [5 marks]

**(a)** Find its inverse and describe the relationship between the two graphs. [3 marks]

$y = \ln(3x)$ implies $x = \frac{e^y}{3}$.

**Answer:** $f^{-1}(x) = \frac{e^x}{3}$; reflection in $y = x$.

**(b)** Find the point on the inverse corresponding to $(\frac{1}{3}, 0)$ on $f$. [2 marks]

An inverse interchanges input and output.

**Answer:** $(0, \frac{1}{3})$.

**7.** Let $f(x) = a\ln x + bx$ for $x > 0$. Its tangent at $x = 1$ is horizontal and $f(1) = 6$. [6 marks]

**(a)** Determine $a$ and $b$. [3 marks]

$f(1) = b = 6$. The derivative is $\frac{a}{x} + b$, so $a + b = 0$.

**Answer:** $a = -6$, $b = 6$.

**(b)** Classify the stationary point at $x = 1$. [3 marks]

$f'' = -\frac{a}{x^2} = \frac{6}{x^2} > 0$.

**Answer:** A local minimum at $(1, 6)$.

**8.** A mathematical cost model is $C(x) = 20 + x - 4\ln x$ for $x > 0$. [6 marks]

**(a)** Find the value of $x$ minimising $C$ and justify that it is a global minimum. [3 marks]

$C' = 1 - \frac{4}{x}$ is negative on $(0, 4)$ and positive on $(4, \infty)$.

**Answer:** $x = 4$.

**(b)** Give the minimum exactly and explain why $x = 0$ cannot be tested as an endpoint. [3 marks]

The domain is open at zero; the function tends to infinity as $x \to 0^+$.

**Answer:** $24 - 4\ln 4$; zero is outside the logarithm domain.

**9.** Distance travelled in a supplied model is $D(t) = 12\ln(t+1)$ metres, $t \geq 0$ seconds. [5 marks]

**(a)** Find the time when $D = 12$ metres. [2 marks]

$\ln(t+1) = 1.$

**Answer:** $t = e - 1$ seconds.

**(b)** If $t+1$ doubles, show that the additional distance is constant and find it. [3 marks]

$12\ln(2(t+1)) - 12\ln(t+1) = 12\ln 2.$

**Answer:** $12\ln 2$ metres.

**10.** A student fits a formula without checking its useful domain. [5 marks]

**(a)** Find the mathematical domain of $M(t) = 50 + 10\ln(t-2)$. [2 marks]

Require $t - 2 > 0.$

**Answer:** $t > 2.$

**(b)** The data were observed only for $3 \leq t \leq 8$ days. Explain two separate cautions about predicting at $t = 1$ and $t = 100$. [3 marks]

Distinguish a mathematical domain restriction from limited empirical validity.

**Answer:** At $t = 1$ the formula is not real; at $t = 100$ it is defined but is a large extrapolation.