Logarithmic functions - Practice test 3

Name: _____ Date: _____

10 questions | 56 marks | 70 minutes

A mixed paper connecting inverse functions, differentiation and supplied mathematical models.

A scientific calculator may be used.

1. Let $f(x) = 2e^{3x}$. [5 marks]

(a) Find its inverse function. [3 marks]

(b) Explain the range of the inverse and evaluate it at $2e$. [2 marks]

2. Apply logarithm laws. [5 marks]

(a) For $x > 1$, expand $\ln\left(\frac{(x-1)^3}{\sqrt{x+2}}\right)$. [3 marks]

(b) Simplify $\log_4 2 + \log_4 8$. [2 marks]

3. Solve $[\ln x]^2 - 3\ln x + 2 = 0$. [5 marks]

(a) State a substitution and find the possible logarithm values. [3 marks]

(b) Find all x and explain why neither is rejected. [2 marks]

4. Consider two exponential expressions. [5 marks]

(a) Solve $e^{2x} + 2e^x - 8 = 0$. [3 marks]

(b) Explain why $e^{2x} + 2e^x + 8 = 0$ has no real solution without using the discriminant. [2 marks]

5. A deposit is modelled by $A(n) = 1000(1.04)^n$ after n complete years. [6 marks]

(a) Extend the model to real values of n and find the time at which A reaches 1500. [3 marks]

(b) Find the first complete year with A at least 1500, justifying the rounding direction. [3 marks]

6. A transformed graph has form $g(x) = a\ln(x-1) + b$. [6 marks]

(a) Use $g(2) = 4$ and $g(1+e) = 7$ to find a and b. [3 marks]

(b) Find its x-intercept exactly and its vertical asymptote. [3 marks]

7. Let $f(x) = \frac{\ln x}{x}$ for $x > 0$. [6 marks]

(a) Find its derivative. [3 marks]

(b) Find and classify its stationary point. [3 marks]

8. Let $f(x) = \ln(1 + x^2)$. [7 marks]

(a) Find the first and second derivatives. [4 marks]

(b) Find the inflection points and verify a change of concavity. [3 marks]

9. A supplied response scale is $S = 5\ln(\frac{Q}{Q_0})$ for $Q, Q_0 > 0$. [5 marks]

(a) Find $\frac{Q}{Q_0}$ when $S = 15$. [2 marks]

(b) If Q doubles, find the exact change in S and explain why it does not depend on Q_0. [3 marks]

10. Two proposed growth models are $F(t) = 10 + 4\ln(t+1)$ and $G(t) = 10 + 2t$ for $t \geq 0$. [6 marks]

(a) Show they have equal initial values but different initial rates. [3 marks]

(b) At what time are their instantaneous rates equal? Does equal rate imply equal value there? [3 marks]