Logarithmic functions - Practice test 2

10 questions | 54 marks | 70 minutes

Emphasis on domains, reflected graphs, exact solutions and logarithmic rates.

Equivalent correct methods earn the corresponding marks.

1. Identify logarithmic bases and values. [4 marks]

(a) Find $b > 0$, $b \neq 1$, if $\log_b 16 = 4$. [2 marks]

$b^4 = 16$ and the base must be positive.

Answer: $b = 2$

(b) Evaluate $\log_{\frac{1}{3}} 27$. [2 marks]

$(\frac{1}{3})^{-3} = 27$.

Answer: -3

2. Use an identity without losing its domain. [5 marks]

(a) State the real domain of $\ln(x^2)$ and simplify it using $|x|$. [3 marks]

x^2 is positive except at zero; $|x|$ is positive on this domain.

Answer: $x \neq 0$; $2\ln|x|$.

(b) Explain why $\ln(x^2) = 2\ln x$ fails at $x = -2$. [2 marks]

$\ln(-2)$ is not real.

Answer: The left side is $\ln 4$ but the right side is undefined over the reals.

3. For $g(x) = \ln(x+4) - \ln(x-1)$: [5 marks]

(a) State the domain. [2 marks]

Both inputs must be positive: $x > -4$ and $x > 1$.

Answer: $x > 1$

(b) Solve $g(x) = \ln 2$. [3 marks]

$\ln(\frac{x+4}{x-1}) = \ln 2$ gives $x + 4 = 2x - 2$. The result is in the domain.

Answer: $x = 6$

4. Solve $4^x - 5 \cdot 2^x + 4 = 0$. [6 marks]

(a) Make a substitution and solve the resulting polynomial. [3 marks]

$4^x = (2^x)^2$, so $(u-1)(u-4) = 0$.

Answer: $u = 2^x > 0$; $u = 1$ or 4.

(b) Give all real values of x and check them in the original equation. [3 marks]

At 0: $1 - 5 + 4 = 0$. At 2: $16 - 20 + 4 = 0$.

Answer: $x = 0$ or 2.

5. Simplify carefully. [5 marks]

(a) Write $\ln(e^3 + e)$ as $k + \ln A$ with k an integer and $A > 0$. [2 marks]

Factor $e^3 + e = e(e^2 + 1)$, then use the product law.

Answer: $1 + \ln(e^2 + 1)$

(b) Solve $\ln(e^{2x} + 1) = \ln 10$. [3 marks]

$e^{2x} + 1 = 10$, hence $e^{2x} = 9$ and $2x = \ln 9$.

Answer: $x = \ln 3$

6. Consider $h(x) = 2 - \ln(5 - x)$. [6 marks]

(a) Find its domain, vertical asymptote and y-intercept. [3 marks]

Require $5 - x > 0$ and substitute zero for the intercept.

Answer: $x < 5$; $x = 5$; $(0, 2 - \ln 5)$.

(b) Find where $h(x) = 2$ and state whether h is increasing or decreasing. [3 marks]

$\ln(5 - x) = 0$ gives $x = 4$. Also $h'(x) = \frac{1}{5 - x} > 0$ on its domain.

Answer: $x = 4$; increasing.

7. Let $f(x) = x\ln x - 2x$ for $x > 0$. [6 marks]

(a) Find the stationary point. [3 marks]

$f' = \ln x - 1 = 0$ at $x = e$; substitute into f.

Answer: $(e, -e)$

(b) Classify it and determine the range. [3 marks]

$f'' = \frac{1}{x} > 0$. The derivative changes from negative to positive and the function tends to infinity as $x \to \infty$.

Answer: A minimum; range $[-e, \infty)$.

8. Let $y = \log_2(x + 3)$. [5 marks]

(a) Find its derivative and domain. [2 marks]

Write the logarithm as $\frac{\ln(x + 3)}{\ln 2}$.

Answer: $\frac{1}{(x + 3)\ln 2}$; $x > -3$.

(b) Find the equation of the normal at $x = 1$. [3 marks]

The point is $(1, 2)$ and tangent gradient $\frac{1}{4\ln 2}$; the normal gradient is its negative reciprocal.

Answer: $y - 2 = -4\ln 2(x - 1)$.

9. A response model is $R(t) = a + b\ln(t+1)$, $t \geq 0$ hours. Its values are supplied. [6 marks]

Observed values

t (hours)	$R(t)$
0	12
2	30

(a) Find exact values of a and b. [3 marks]

$R(0) = a = 12$. Then $30 = 12 + b\ln 3$.

Answer: $a = 12$, $b = \frac{18}{\ln 3}$.

(b) Predict $R(8)$ and give one limitation of the prediction. [3 marks]

$\ln 9 = 2\ln 3$, so $R(8) = 12 + 36 = 48$. Only times 0 and 2 were supplied.

Answer: 48; the model may not remain valid beyond the observed interval.

10. A logarithm with base less than one reverses order. [6 marks]

(a) Solve $\log_{\frac{1}{2}} x > 2$ over the reals. [3 marks]

The function is decreasing. Values greater than 2 correspond to positive inputs smaller than $(\frac{1}{2})^2$.

Answer: $0 < x < \frac{1}{4}$.

(b) Solve $\ln(x-2) < 0$. [3 marks]

The input must satisfy $0 < x - 2 < 1$.

Answer: $2 < x < 3$.