Logarithmic functions - Practice test 1

Name: _____ Date: _____

10 questions | 50 marks | 70 minutes

A balanced paper moving from inverse relationships to equations, graphs, derivatives and applications.

A scientific calculator may be used.

1. Use logarithms as exponents. [4 marks]

(a) Evaluate $\log_2\left(\frac{1}{32}\right)$ and explain the sign. [2 marks]

(b) Solve $\log_3 x = \frac{5}{2}$ exactly. [2 marks]

2. Apply laws of logarithms. [4 marks]

(a) For $x, y > 0$, condense $2\ln x + \ln 3 - \ln y$. [2 marks]

(b) Give a numerical counterexample to $\ln(a+b) = \ln a + \ln b$. [2 marks]

3. An unfamiliar logarithm base is needed. [4 marks]

(a) Show that $\log_7 20 = \frac{\ln 20}{\ln 7}$. [2 marks]

(b) Evaluate $\log_7 20$ to three decimal places. [2 marks]

4. Solve equations while keeping the original domains. [5 marks]

(a) Solve $\ln(x-2) = \ln 7$. [2 marks]

(b) Solve $\ln(x-1) + \ln(x+1) = \ln 15$. [3 marks]

5. Solve exponential equations exactly. [5 marks]

(a) Solve $4 \cdot 3^{2x} = 28$. [2 marks]

(b) Find all real solutions of $e^{2x} - 7e^{x} + 12 = 0$. [3 marks]

6. Work with natural logarithms. [5 marks]

(a) Simplify $\ln(5e^{2}) - \ln 5$ exactly. [2 marks]

(b) Solve $2\ln x = \ln(2x + 3)$. [3 marks]

7. Let $f(x) = \ln(x-2) + 3$. [6 marks]

(a) State the domain and vertical asymptote. [2 marks]

(b) Find the x-intercept and the inverse function. [4 marks]

8. Consider $y = \ln(5 + 2x)$. [5 marks]

(a) Differentiate and state the real domain. [2 marks]

(b) Find the tangent line at $x = 0$. [3 marks]

9. A training score follows $S(t) = 20 + 8\ln(t+1)$ for $t \geq 0$ days. [6 marks]

(a) Find when $S(t) = 36$. Give an exact answer and then three significant figures. [3 marks]

(b) Find the instantaneous rate at day 3 and explain its units. [3 marks]

10. A supplied intensity scale is $L = 10\log_{10}\left(\frac{I}{I_0}\right)$, where $I, I_0 > 0$. [6 marks]

(a) Find the intensity ratio associated with a 20-unit rise in L. [3 marks]

(b) Explain why the model cannot accept $I = 0$, and whether a negative L is possible. [3 marks]