# Logarithmic functions - Practice test 1

10 questions | 50 marks | 70 minutes

A balanced paper moving from inverse relationships to equations, graphs, derivatives and applications.

Equivalent correct methods earn the corresponding marks.

**1.** Use logarithms as exponents. [4 marks]

**(a)** Evaluate $\log_2\left(\frac{1}{32}\right)$ and explain the sign. [2 marks]

$2^{-5} = \frac{1}{32}$.

**Answer:** $-5$; a negative exponent gives a positive reciprocal.

**(b)** Solve $\log_3 x = \frac{5}{2}$ exactly. [2 marks]

$x = 3^{\frac{5}{2}} = 9\sqrt{3}$.

**Answer:** $x = 9\sqrt{3}$

**2.** Apply laws of logarithms. [4 marks]

**(a)** For $x, y > 0$, condense $2\ln x + \ln 3 - \ln y$. [2 marks]

The power law gives $\ln(x^2)$; product and quotient laws combine the inputs.

**Answer:** $\ln\left(\frac{3x^2}{y}\right)$

**(b)** Give a numerical counterexample to $\ln(a+b) = \ln a + \ln b$. [2 marks]

The right side is $\ln 1 + \ln 1 = 0$, whereas $\ln 2 > 0$.

**Answer:** For $a = b = 1$, $\ln 2 \neq 0$.

**3.** An unfamiliar logarithm base is needed. [4 marks]

**(a)** Show that $\log_7 20 = \frac{\ln 20}{\ln 7}$. [2 marks]

Set $u = \log_7 20$ and take natural logarithms.

**Answer:** $7^u = 20 \Rightarrow u\ln 7 = \ln 20$.

**(b)** Evaluate $\log_7 20$ to three decimal places. [2 marks]

$\frac{\ln 20}{\ln 7} \approx 1.539502$; to three decimal places this is $1.540$.

**Answer:** $1.540$

**4.** Solve equations while keeping the original domains. [5 marks]

**(a)** Solve $\ln(x-2) = \ln 7$. [2 marks]

The domain is $x > 2$. Equal logarithms have equal positive inputs, so $x - 2 = 7$.

**Answer:** $x = 9$

**(b)** Solve $\ln(x-1) + \ln(x+1) = \ln 15$. [3 marks]

The domain is $x > 1$. Then $x^2 - 1 = 15$ gives $x = \pm 4$; only $4$ is valid.

**Answer:** $x = 4$

**5.** Solve exponential equations exactly. [5 marks]

**(a)** Solve $4 \cdot 3^{2x} = 28$. [2 marks]

$3^{2x} = 7$, then $2x\ln 3 = \ln 7$.

**Answer:** $x = \frac{\ln 7}{2\ln 3}$

**(b)** Find all real solutions of $e^{2x} - 7e^x + 12 = 0$. [3 marks]

Put $u = e^x > 0$. $(u-3)(u-4) = 0$; both roots are positive.

**Answer:** $x = \ln 3$ or $x = \ln 4$

**6.** Work with natural logarithms. [5 marks]

**(a)** Simplify $\ln(5e^2) - \ln 5$ exactly. [2 marks]

Use the quotient law: $\ln(e^2) = 2$.

**Answer:** $2$

**(b)** Solve $2\ln x = \ln(2x + 3)$. [3 marks]

Domain $x > 0$. The equation becomes $x^2 - 2x - 3 = 0$, giving $3, -1$. Reject $-1$.

**Answer:** $x = 3$

**7.** Let $f(x) = \ln(x - 2) + 3$. [6 marks]

**(a)** State the domain and vertical asymptote. [2 marks]

The logarithm input must be positive and tends to zero at $x = 2$.

**Answer:** $x > 2$; asymptote $x = 2$.

**(b)** Find the x-intercept and the inverse function. [4 marks]

At $y = 0$, $\ln(x - 2) = -3$. For the inverse solve $y - 3 = \ln(x - 2)$ for $x$, then swap names.

**Answer:** $(2 + e^{-3}, 0)$; $f^{-1}(x) = 2 + e^{x-3}$.

**8.** Consider $y = \ln(5 + 2x)$. [5 marks]

**(a)** Differentiate and state the real domain. [2 marks]

Apply input derivative divided by input; require $5 + 2x > 0$.

**Answer:** $y' = \frac{2}{5+2x}$; $x > -\frac{5}{2}$.

**(b)** Find the tangent line at $x = 0$. [3 marks]

The point is $(0, \ln 5)$ and the gradient is $\frac{2}{5}$.

**Answer:** $y = \ln 5 + \frac{2}{5}x$.

**9.** A training score follows $S(t) = 20 + 8\ln(t+1)$ for $t \geq 0$ days. [6 marks]

**(a)** Find when $S(t) = 36$. Give an exact answer and then three significant figures. [3 marks]

$8\ln(t+1) = 16$, so $t + 1 = e^2$.

**Answer:** $t = e^2 - 1 \approx 6.39$ days.

**(b)** Find the instantaneous rate at day 3 and explain its units. [3 marks]

$S'(t) = \frac{8}{t+1}$, so $S'(3) = 2$.

**Answer:** $2$ score units per day.

**10.** A supplied intensity scale is $L = 10\log_{10}\left(\frac{I}{I_0}\right)$, where $I, I_0 > 0$. [6 marks]

**(a)** Find the intensity ratio associated with a 20-unit rise in $L$. [3 marks]

$20 = 10\log_{10}\left(\frac{I_2}{I_1}\right)$; exponentiating gives $10^2$.

**Answer:** $\frac{I_2}{I_1} = 100$.

**(b)** Explain why the model cannot accept $I = 0$, and whether a negative $L$ is possible. [3 marks]

A positive input less than one has a negative base-10 logarithm.

**Answer:** $I = 0$ gives no real logarithm; negative $L$ is possible when $0 < I < I_0$.