Logarithmic modelling

6 questions | 17 marks

Equivalent correct methods earn the corresponding marks.

1. For $P(t) = 15 + 6\ln(t+1)$, $t \geq 0$ weeks, find $P(0)$ and $P(3)$. [3 marks]

Substitute the times; $\ln 1 = 0$.

Answer: 15 and $15 + 6\ln 4 \approx 23.3$

2. Find when the model $R(t) = 10 + 5\ln(t+1)$ first reaches 20, with t in hours. [3 marks]

$\ln(t+1) = 2$, then exponentiate and subtract 1.

Answer: $t = e^2 - 1 \approx 6.39$ hours

3. For $R(t) = 10 + 5\ln(t+1)$ with $t \geq 0$ measured in hours, find and interpret $R'(4)$. [3 marks]

$R'(t) = \frac{5}{t+1}$, then substitute $t = 4$.

Answer: 1 response unit per hour.

4. A scale is $S = 20\log_{10}(\frac{A}{A_0})$. What amplitude ratio gives a rise of 12 units? [3 marks]

$12 = 20\log_{10}(\frac{A_2}{A_1})$. Exponentiate base 10.

Answer: $10^{0.6} \approx 3.98$

5. The model $F(x) = a + b\ln x$ gives $F(1) = 8$ and $F(e^2) = 18$. Find a, b. [3 marks]

$\ln 1 = 0$ gives $a = 8$. Then $18 = 8 + 2b$.

Answer: $a = 8$, $b = 5$

6. A score $30 + 12\ln(t+1)$ is fitted using data from days 0-10. Explain one reason not to trust its prediction for day 1000. [2 marks]

The formula grows without bound. A real learning score may have a maximum or change its pattern.

Answer: The prediction extrapolates far beyond the measured interval and may ignore a real score limit.