Differentiating logarithms and composite logarithms

6 questions | 16 marks

Equivalent correct methods earn the corresponding marks.

1. Differentiate $\ln(7x)$, retaining its domain. [2 marks]

$\frac{u'}{u} = \frac{7}{7x} = \frac{1}{x}$.

Answer: $\frac{1}{x}$, $x > 0$

2. Differentiate $\log_3(2x+5)$. [2 marks]

Use change of base, then apply the chain rule.

Answer: $\frac{2}{(2x+5)\ln 3}$, $x > -\frac{5}{2}$

3. Differentiate $(\ln x)^2$. [2 marks]

The outer square contributes $2\ln x$ and the inner derivative is $\frac{1}{x}$.

Answer: $\frac{2\ln x}{x}$, $x > 0$

4. Differentiate $y = \ln(x^2 - 9)$ and state the domain. [3 marks]

Solve $x^2 - 9 > 0$ before applying the chain rule.

Answer: $y' = \frac{2x}{x^2-9}$; $x < -3$ or $x > 3$

5. Find the stationary point of $f(x) = \ln x - \frac{x}{2}$ and classify it. [4 marks]

$f' = \frac{1}{x} - \frac{1}{2} = 0$ gives $x = 2$.

$f'' = -\frac{1}{x^2} < 0$.

Answer: $(2, \ln 2 - 1)$, a local maximum.

6. Find the tangent to $y = \ln(2x-1)$ at $x = 1$. [3 marks]

The point is $(1, 0)$ and the gradient is $\frac{2}{2-1} = 2$.

Answer: $y = 2x - 2$