Solving exponential equations

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. Solve $3^{2x-1} = 27$. [2 marks]

$27 = 3^3$, so $2x - 1 = 3$.

Answer: $x = 2$

2. Solve $4 \cdot 5^x = 28$ exactly. [2 marks]

$5^x = 7$, then take logarithms.

Answer: $x = \frac{\ln 7}{\ln 5}$

3. Solve $e^{2x} - 5e^x + 6 = 0$. [3 marks]

Let $u = e^x$. $(u-2)(u-3) = 0$ and both roots are positive.

Answer: $x = \ln 2$ or $x = \ln 3$

4. A quantity is $Q(t) = 80e^{-0.12t}$, with t in hours. When is $Q = 20$? [3 marks]

$e^{-0.12t} = \frac{1}{4}$. Thus $-0.12t = -\ln 4$.

Answer: $t = \frac{\ln 4}{0.12} \approx 11.6$ hours

5. Solve $2^x = 3^{x-1}$. [3 marks]

$x\ln 2 = (x-1)\ln 3$. Collect the terms in x.

Answer: $x = \frac{\ln 3}{\ln 3 - \ln 2} \approx 2.710$

6. Explain whether $7e^{-x} + 2 = 1$ has a real solution. [2 marks]

It would require $e^{-x} = -\frac{1}{7}$, but real exponentials are positive.

Answer: No real solution.