Logarithm laws and change of base

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

1. For $a, b > 0$, expand $\ln\left(\frac{5a^2}{b}\right)$. [3 marks]

Use product, power and quotient laws; the factors are positive.

Answer: $\ln 5 + 2\ln a - \ln b$

2. For $x > 0$, condense $3\log_2 x + \log_2 5$. [2 marks]

$3\log_2 x = \log_2 (x^3)$, then combine the product.

Answer: $\log_2 (5x^3)$

3. Find $\log_5 12$ to three decimal places. [2 marks]

$\log_5 12 = \frac{\ln 12}{\ln 5} \approx 1.543959$.

Answer: 1.544

4. Simplify $\log_2 48 - \log_2 3$. [2 marks]

$\log_2 \left(\frac{48}{3}\right) = \log_2 16 = 4$.

Answer: 4

5. Explain why replacing $\ln(x^2)$ by $2\ln x$ requires care. [3 marks]

The original domain is $x \neq 0$. On that domain, $\ln(x^2) = 2\ln|x|$.

Answer: The replacement is valid for $x > 0$; $\ln(x^2)$ is also defined for $x < 0$.

6. If $\ln a = 2$ and $\ln b = -1$, find $\ln\left(\frac{a^2}{b^3}\right)$. [2 marks]

$2\ln a - 3\ln b = 4 + 3 = 7$.

Answer: 7