Differentiation practice test 5: interpretation and decisions

Name: _____ Date: _____

10 questions | 64 marks | 75 minutes

Original chapter assessment with linked reasoning and applications. Show working, state relevant domains and retain exact values until a numerical answer is requested.

A scientific calculator may be used.

1. The table gives values and derivatives of two functions at $x = 2$. [5 marks]

x	$u(x)$	$u'(x)$	$v(x)$	$v'(x)$
2	3	-1	5	4

(a) Find $(uv)'(2)$. [2 marks]

(b) Find $\left(\frac{u}{v}\right)'(2)$. [3 marks]

2. Let $f(x) = \sqrt{9 - x^2}$. [6 marks]

(a) State the real function domain and find its derivative. [3 marks]

(b) Find the tangent at zero and explain why the endpoints are not stationary points. [3 marks]

3. Let $g(x) = x^2 e^{-x}$ for real x. In part (b), restrict to $0 \le x \le 4$. [7 marks]

(a) Find and classify the stationary points. [4 marks]

(b) Find the absolute maximum on the interval. [3 marks]

4. For $a > 0$, the identity $a^x = e^{x\ln a}$ is supplied. [5 marks]

(a) Derive a formula for the derivative of a^x. [3 marks]

(b) Find the tangent to $y = 3^x$ at zero. [2 marks]

5. Analyse $f(x) = x^5 - 5x$. [6 marks]

(a) Find and classify the stationary points. [4 marks]

(b) Find and justify its inflection point. [2 marks]

6. Let $h(x) = \frac{1}{1+x^2}$. [7 marks]

(a) Find its first derivative. [2 marks]

(b) Show that $h'' = \frac{6x^2-2}{(1+x^2)^3}$, then find the inflection points. [5 marks]

7. A height is $H(t) = 10 + 4t - t^2$ metres for $0 \le t \le 4$ seconds. [6 marks]

(a) Find its average rate from 0 to 2 seconds. [2 marks]

(b) Find when its instantaneous rate equals that average. [2 marks]

(c) Find its maximum height. [2 marks]

8. A rectangular enclosure is divided into three equal pens by two internal fences parallel to its width W. There is 120 m of fencing for the boundary and the two dividers. [6 marks]

(a) Express length L and total area in terms of W. [3 marks]

(b) Find dimensions giving greatest total area. [3 marks]

9. An open cylindrical container has volume 36π cubic centimetres. It has a base but no lid. [7 marks]

(a) Write its surface area as a function of radius r. [3 marks]

(b) Find the dimensions minimising material area. [4 marks]

10. A damped displacement model is $s(t) = e^{-t}\cos(2t)$ metres, $t \geq 0$. [9 marks]

(a) Find velocity and acceleration. [4 marks]

(b) Is the particle speeding up at zero? [2 marks]

(c) Find its first positive rest time to 3 significant figures. [3 marks]