Differentiation practice test 2

Name: _____ Date: _____

10 questions | 61 marks | 75 minutes

An original progressively demanding chapter paper. Show analytical working; a scientific calculator may be used for numerical values. Diagrams and data needed for questions are supplied.

A scientific calculator may be used.

1. Let $f(x) = 2\sqrt{x} + \frac{3}{x}$. [5 marks]

(a) Differentiate and state the derivative domain. [3 marks]

(b) Find the gradient at $x = 1$. [2 marks]

2. Let $g(x) = xe^{-2x}$. [6 marks]

(a) Find the derivative. [3 marks]

(b) Find and classify its stationary point. [3 marks]

3. Let $h(x) = \frac{2x-3}{x^2+1}$. [5 marks]

(a) Find the derivative. [3 marks]

(b) Find the tangent at zero. [2 marks]

4. Consider $y = (1-3x^2)^5$. [5 marks]

(a) Differentiate. [2 marks]

(b) Find the second derivative at zero. [3 marks]

5. A height model is $H(t) = 4 + 3\cos(\frac{\pi t}{6})$ metres. [6 marks]

(a) Find its rate of change at 3 seconds. [3 marks]

(b) Find its second derivative at zero. [3 marks]

6. A differentiable function has $f'(x) = x^2(x-2)$. [7 marks]

(a) Find the stationary inputs and classify them. [4 marks]

(b) Find the inputs where concavity changes. [3 marks]

7. Let $r(x) = \frac{x^2}{x-1}$. [9 marks]

(a) State its vertical and oblique asymptotes. [3 marks]

(b) Find and classify the stationary points. [4 marks]

(c) Is $x = 1$ an inflection point? [2 marks]

8. Temperature is modelled by $T(t) = 30 + 120e^{-0.2t}$ degrees Celsius, with time in minutes. [6 marks]

(a) Find the initial temperature and initial cooling rate. [3 marks]

(b) When does temperature reach 75 degrees Celsius? [3 marks]

9. A rectangle has area 64 square metres. [5 marks]

(a) Express its perimeter using one positive side x. [2 marks]

(b) Find the least perimeter and justify it. [3 marks]

10. Position is $s(t) = \frac{t^3}{3} - 4t^2 + 15t$ metres, $0 \leq t \leq 8$ seconds. [7 marks]

(a) Find direction-change times. [3 marks]

(b) Find the displacement and total distance. [4 marks]