Marking guide | Differentiation and applications

Differentiation practice test 1

10 questions | 55 marks | 75 minutes

An original progressively demanding chapter paper. Show analytical working; a scientific calculator may be used for numerical values. Diagrams and data needed for questions are supplied.

Equivalent correct methods earn the corresponding marks.

1. Let $f(x) = 2x^3 - 3x^2 + 4$. [5 marks]

(a) Find $f'(x)$. [2 marks]

Apply the power rule term by term.

Answer: $6x^2 - 6x$

(b) Find the equation of the tangent at $x = 2$. [3 marks]

The point is $(2, 8)$ and $f'(2) = 12$.

Use $y - 8 = 12(x - 2)$.

Answer: $y = 12x - 16$

2. Let $g(x) = (x^2 + 1)(x - 2)$. [5 marks]

(a) Differentiate in product form, then simplify. [3 marks]

$g' = 2x(x - 2) + (x^2 + 1)$.

Expand and collect.

Answer: $g'(x) = 3x^2 - 4x + 1$

(b) Find the tangent at $x = 0$. [2 marks]

The point is $(0, -2)$ and the gradient is 1.

Answer: $y = x - 2$

3. Let $h(x) = (2x-1)^4$. [5 marks]

(a) Find the derivative. [2 marks]

Use the outer power rule and multiply by inner derivative 2.

Answer: $h'(x) = 8(2x-1)^3$

(b) Find the stationary point and classify it. [3 marks]

The derivative is zero at $\frac{1}{2}$.

Its sign changes from negative to positive.

Answer: Minimum $(\frac{1}{2}, 0)$.

4. Let $r(x) = \frac{x^2+1}{x+1}$. [5 marks]

(a) State the domain and find the derivative. [3 marks]

Use $\frac{2x(x+1)-(x^2+1)}{(x+1)^2}$.

Answer: $x \neq -1$; $r' = \frac{x^2+2x-1}{(x+1)^2}$

(b) Find the gradient at $x = 1$. [2 marks]

The numerator is 2 and denominator 4.

Answer: $\frac{1}{2}$

5. Consider $y = 3e^{2x}$. [4 marks]

(a) Find the derivative. [2 marks]

Multiply by the exponent derivative 2.

Answer: $6e^{2x}$

(b) Find the tangent at the y-intercept. [2 marks]

At zero, height is 3 and gradient is 6.

Answer: $y = 6x + 3$

6. For radian input, $f(x) = 2\sin(3x) - \cos x$. [4 marks]

(a) Differentiate. [2 marks]

Differentiate sine with its inner factor.

The negative cosine term gives positive sine.

Answer: $f' = 6\cos(3x) + \sin x$

(b) Find the tangent at $x = 0$. [2 marks]

The point is $(0, -1)$ and gradient is 6.

Answer: $y = 6x - 1$

7. Analyse $F(x) = x^3 - 6x^2 + 9x$. [7 marks]

(a) Find and classify both stationary points. [4 marks]

$F' = 3(x-1)(x-3)$ and $F'' = 6x - 12$.

Evaluate signs and heights.

Answer: Maximum $(1, 4)$; minimum $(3, 0)$.

(b) Find its inflection point, justifying the change. [3 marks]

The second derivative changes from negative to positive at 2.

Substitution gives height 2.

Answer: $(2, 2)$

8. A medicine model is $M(t) = 200e^{-0.05t}$ mg, with t in hours. [6 marks]

(a) Find the rate at $t = 10$, to 3 significant figures. [3 marks]

$M' = -10e^{-0.05t}$.

Evaluate at 10.

Answer: $-10e^{-0.5} \approx -6.07$ mg/h

(b) Find its half-life. [3 marks]

$100 = 200e^{-0.05t}$, so $-0.05t = -\ln 2$.

Answer: $20\ln 2 \approx 13.9$ h

9. A rectangle has perimeter 40 cm. Let one side be x cm. [5 marks]

(a) Form an area model and state the physical domain. [2 marks]

The other side is $20 - x$; both sides must be positive.

Answer: $A = x(20 - x)$, $0 < x < 20$

(b) Find and justify the greatest area. [3 marks]

$A' = 20 - 2x = 0$ at 10; $A'' = -2 < 0$.

Answer: $100\ \text{cm}^2$ at sides 10 cm and 10 cm

10. A particle moves on a line with $s(t) = t^3 - 9t^2 + 24t$ metres for $0 \le t \le 6$ seconds. [9 marks]

(a) Find the rest times and the interval of negative motion. [3 marks]

$v = 3(t - 2)(t - 4)$; use a sign chart.

Answer: Rest at 2 s and 4 s; negative for $2 < t < 4$.

(b) Find displacement and total distance. [4 marks]

Positions at times $0, 2, 4, 6$ are $0, 20, 16, 36$.

Displacement is the last minus first; distance is $20 + 4 + 20$.

Answer: Displacement 36 m; distance 44 m.

(c) Is the particle speeding up at 5 seconds? [2 marks]

At 5, velocity is 9 m/s and acceleration is 12 m/s squared.

Both signs are positive.

Answer: Yes.