Rates and motion applications

6 questions | 18 marks

Equivalent correct methods earn the corresponding marks.

1. For $s(t)=2t^2-5t+1$ metres, with t in seconds, find velocity and acceleration. [2 marks]

Differentiate position once for velocity.

Differentiate again for acceleration.

Answer: $v=4t-5$ m/s; $a=4$ m/s squared.

2. For $s(t)=t^3-3t$ metres on $t\geq 0$ seconds, find the rest time and whether direction changes. [3 marks]

The velocity is $3(t-1)(t+1)$.

On the physical domain it changes sign at 1.

Answer: $t=1$ s; direction changes from negative to positive.

3. For $s(t)=t^3-6t^2+9t$ metres on $0\leq t\leq 4$ seconds, find displacement and total distance. [4 marks]

The reversal times are 1 and 3.

Positions at $0, 1, 3, 4$ are $0, 4, 0, 4$.

Add absolute position changes for distance.

Answer: Displacement 4 m; distance 12 m.

4. For velocity $v(t) = t^2 - 4t + 3$ metres per second, with t in seconds, find the speed and its instantaneous rate of change at $t = 2$ seconds. [3 marks]

The velocity at 2 is -1.

Acceleration is $2t - 4$, which is zero at 2.

Answer: Speed 1 m/s; acceleration is zero at that instant, so the instantaneous rate of change of speed is zero.

5. A cube has side length increasing at 0.2 cm/s. Find the volume rate when the side is 3 cm. [3 marks]

Use $V = x^3$ and differentiate with respect to time.

Then $\frac{dV}{dt} = 3x^2\frac{dx}{dt} = 3(9)(0.2)$.

Answer: $5.4\ \text{cm}^3/\text{s}$

6. A cost model is $C(q) = 0.02q^2 + 5q + 100$ dollars. Find and interpret marginal cost at 50 units. [3 marks]

Differentiate with respect to quantity: $C'(q) = 0.04q + 5$.

At 50, this is 7; it is a local rate, not the exact finite cost of the next unit.

Answer: 7 per additional unit approximately.