Optimisation

6 questions | 24 marks

Equivalent correct methods earn the corresponding marks.

1. A rectangle has perimeter 28 m. Find its maximum area. [3 marks]

The sides are x and $14-x$.

Maximise $14x-x^2$, whose derivative vanishes at 7.

Answer: $49\ \text{m}^2$ for a 7 m square.

2. A three-sided pen beside a wall uses 80 m of fencing. Find optimal dimensions. [3 marks]

Maximise $x(80-2x)$.

The derivative $80-4x$ is zero at 20.

Answer: 20 m perpendicular to the wall and 40 m parallel; area $800\ \text{m}^2$.

3. Squares of side x are cut from each corner of a 12 cm square to form an open box. Find the maximum volume. [4 marks]

The volume is $V=x(12-2x)^2$, with $0<x<6$.

Its derivative is $12(x-2)(x-6)$.

The interior maximum is at 2; both limiting endpoint volumes are zero.

Answer: $128\ \text{cm}^3$ at $x=2$ cm.

4. A product costs $C(x) = x^2 + 100$ dollars to make $x > 0$ units. Minimise average cost per unit. [4 marks]

Average cost is $x + \frac{100}{x}$.

Its derivative is $1 - \frac{100}{x^2}$, zero at 10.

The derivative changes from negative to positive.

Answer: 20 per unit at $x = 10$.

5. A point (x, x^2) lies on a parabola. Find its least distance from $(0, 2)$. [5 marks]

Minimise the square of distance: $D^2 = x^2 + (x^2 - 2)^2 = x^4 - 3x^2 + 4$.

The stationary inputs are 0 and $\pm\sqrt{\frac{3}{2}}$.

The smaller squared distance is $\frac{7}{4}$; take its square root.

Answer: $\frac{\sqrt{7}}{2}$, at $x = \pm\sqrt{\frac{3}{2}}$.

6. A closed cylinder has total surface area 24π square centimetres. Find its largest volume. [5 marks]

The constraint gives $h = \frac{12}{r} - r$, where $0 < r < \sqrt{12}$.

The volume is $\pi(12r - r^3)$.

Its derivative vanishes at $r = 2$; the endpoint volumes tend to zero.

Answer: 16π cm^3 at radius 2 cm and height 4 cm.