Curve analysis

6 questions | 17 marks

Equivalent correct methods earn the corresponding marks.

1. Find all intercepts of $y = x^3 - 4x$. [2 marks]

Factor as $x(x-2)(x+2)$.

Set each factor to zero.

Answer: x-intercepts $(-2, 0), (0, 0), (2, 0)$; y-intercept $(0, 0)$.

2. State increasing intervals for $y = x^3 - 3x$. [2 marks]

The derivative is $3(x^2 - 1)$.

It is positive when $|x| > 1$.

Answer: $(-\infty, -1)$ and $(1, \infty)$.

3. For $y = x^4 - 2x^2$, find all stationary coordinates. [3 marks]

The derivative is $4x(x^2 - 1)$.

Evaluate the original function at $-1, 0, 1$.

Answer: $(-1, -1), (0, 0), (1, -1)$.

4. Find the vertical and oblique asymptotes of $y = x + \frac{1}{x}$. [2 marks]

The reciprocal term is undefined at zero.

It tends to zero as $|x|$ becomes large.

Answer: $x = 0$ and $y = x$.

5. Sketch $y = e^{-x}$ by stating its intercept, monotonicity, concavity and asymptote. [4 marks]

The first derivative is negative and the second positive.

The exponential remains positive and tends to zero as x increases.

Answer: Intercept $(0, 1)$; decreasing; concave up; horizontal asymptote $y = 0$.

6. For $f(x) = x^3 - 3x$ on $[-2, 2]$, find the absolute maximum and minimum. [4 marks]

Check stationary inputs $-1, 1$ and endpoints $-2, 2$.

The values are respectively $2, -2, -2, 2$.

Answer: Maximum 2 at $x = -1, 2$; minimum -2 at $x = -2, 1$.