Second derivatives and concavity

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. Find $f''(x)$ for $f(x) = x^5 - 2x^3 + x$. [2 marks]

Differentiate to obtain $5x^4 - 6x^2 + 1$.

Differentiate once more.

Answer: $20x^3 - 12x$

2. State the concavity of $y = -2x^2 + 3x + 4$. [2 marks]

The first derivative is $-4x + 3$.

The second derivative is $-4 < 0$.

Answer: Concave down for all real x.

3. For $f(x) = x^4 - 4x^2$, find the intervals of concavity. [3 marks]

The second derivative is $12x^2 - 8$.

Solve $12x^2 - 8 = 0$ and test signs.

Answer: Up for $x < -\sqrt{\frac{2}{3}}$ or $x > \sqrt{\frac{2}{3}}$; down between these values.

4. Give an example of a function that decreases and is concave up for $x > 0$. [3 marks]

Its first derivative is $-e^{-x} < 0$.

Its second derivative is $e^{-x} > 0$.

Answer: $f(x) = e^{-x}$ works.

5. A population model is $P(t) = 100e^{0.2t}$. Find $P''(0)$. [2 marks]

The first derivative is $20e^{0.2t}$.

The second is $4e^{0.2t}$.

Answer: 4 people per time-unit squared

6. If $f''(x) > 0$, does it follow that $f'(x) > 0$? Explain. [3 marks]

For $f(x) = e^{-x}$ the second derivative is positive.

Its first derivative is negative.

Answer: No. It says the gradient is increasing, not necessarily positive.