Counting and statistics - Reasoning test 4

10 questions | 51 marks | 70 minutes

Explain methods, reverse calculations and challenge misleading conclusions. Supplied normal values may be used without further tables.

Equivalent correct methods earn the corresponding marks.

1. A code has 3 letters chosen from A-E, followed by 2 digits 0-9. Repetition is allowed. [5 marks]

(a) Count the codes. [2 marks]

$5^3 \times 10^2 = 12500.$

Answer: 12,500.

(b) How many codes have different letters and different digits? [3 marks]

$5 \times 4 \times 3 \times 10 \times 9 = 5400.$

Answer: 5,400.

2. Six cards show the letters B,A,N,A,N,A. [6 marks]

(a) Count distinct rows of all six cards. [3 marks]

A repeats 3 times and N repeats 2 times.

$\frac{6!}{3!\,2!} = 60.$

Answer: 60.

(b) Count distinct rows that begin and end with A. [3 marks]

Fix an A at each end.

The middle four letters A,B,N,N have $\frac{4!}{2!} = 12$ orders.

Answer: 12.

3. A club has n members. It can choose a president and a different secretary in 56 ways. [5 marks]

(a) Form an equation and find n. [3 marks]

$n^2 - n - 56 = (n-8)(n+7) = 0$.

Reject -7 because membership is positive.

Answer: $n(n-1) = 56$, so $n = 8$.

(b) How many unordered pairs can be chosen from the club? [2 marks]

$\binom{8}{2} = \frac{56}{2} = 28$.

Answer: 28.

4. Consider $(x-2)^5$. [5 marks]

(a) Find the coefficient of x^2. [3 marks]

$\binom{5}{2} \times (-2)^3 = 10 \times (-8) = -80$.

Answer: -80.

(b) Without expanding, find the sum of its coefficients. [2 marks]

Substitute $x = 1$: $(1-2)^5 = -1$.

Answer: -1.

5. A sports survey asks only spectators at a basketball final whether the school should build another basketball court. [5 marks]

(a) Explain why the sample may be biased. [2 marks]

Interest in the sport affects inclusion in the sample.

Answer: Basketball spectators may value a court more highly than the full student population.

(b) Would selecting a random 100 spectators solve the population-coverage problem? Explain. [3 marks]

Random selection within spectators does not include students who are not spectators.

The sampling frame still differs from the target population.

Answer: No.

6. A sample of 8 has mean 12. Seven observed values sum to 77. [4 marks]

(a) Find the missing observation. [2 marks]

Required total $8 \times 12 = 96$.

Missing value $96 - 77 = 19$.

Answer: 19.

(b) If 19 is replaced by 3, find the new mean and describe the direction of change. [2 marks]

The total falls by 16, so the mean falls by $\frac{16}{8} = 2$.

Answer: Mean 10; it decreases by 2.

7. Data A are 0,2,4,6,8,10,12,14. Data B are 0,2,4,5,9,10,12,14. [6 marks]

(a) Find the five-number summary of each using median-of-halves. [3 marks]

$Q_1 = \frac{2+4}{2} = 3$; median A $= \frac{6+8}{2} = 7$, B $= \frac{5+9}{2} = 7$; $Q_3 = \frac{10+12}{2} = 11$.

Answer: Both have $(0, 3, 7, 11, 14)$.

(b) Do these equal summaries guarantee equal standard deviations? Explain. [3 marks]

A and B have different distances from the mean even though selected quantiles match.

A has sample variance 24; B has sample variance $\frac{174}{7} \approx 24.857$.

Answer: No.

8. A set of measurements has sample mean 18°C and $s = 2$°C. A new scale reports $F = 1.8C + 32$. [5 marks]

(a) Find the new mean. [2 marks]

$1.8 \times 18 + 32 = 64.4$.

Answer: 64.4°F.

(b) Find the new standard deviation and explain why 32 does not affect it. [3 marks]

$1.8 \times 2 = 3.6$.

Adding a constant does not alter deviations from the mean.

Answer: 3.6°F.

9. X is normally modelled with mean 75 and sd 10. Supplied $\Phi(1.2) = 0.88493$, $\Phi(-0.8) = 0.21186$. [5 marks]

(a) Find $P(67 < X < 87)$. [3 marks]

Standardized bounds -0.8 and 1.2.

Subtract $0.88493 - 0.21186$.

Answer: 0.67307.

(b) Find the expected number outside this interval in a sample of 200. [2 marks]

Outside proportion $1 - 0.67307 = 0.32693$.

$200 \times 0.32693 = 65.386$.

Answer: 65.386, approximately 65.

10. A normal model has 10th percentile 41.03 and 90th percentile 58.97. The relevant standard-normal cutoffs are ± 1.28155. [5 marks]

(a) Find the mean. [2 marks]

The two percentiles are symmetric, so their midpoint is $\frac{41.03 + 58.97}{2} = 50$.

Answer: 50.

(b) Estimate the standard deviation. [3 marks]

$58.97 = 50 + 1.28155\sigma$.

$\sigma = \frac{8.97}{1.28155} \approx 6.9993$.

Answer: About 7.00.