Counting and statistics - Applications test 3

10 questions | 52 marks | 70 minutes

Use counting and statistics to plan an event and judge operational data. State what each calculation counts and qualify the conclusions.

Equivalent correct methods earn the corresponding marks.

1. A festival program has one music act, one demonstration and one food stall. There are 5 music acts,4 demonstrations and 3 stalls. [5 marks]

(a) Count programs when all combinations are available. [2 marks]

$5 \times 4 \times 3 = 60$.

Answer: 60.

(b) One music act can only be paired with 2 of the demonstrations. All other combinations remain available. Count the programs. [3 marks]

The restricted act gives $1 \times 2 \times 3 = 6$.

The other 4 acts give $4 \times 4 \times 3 = 48$.

Add disjoint cases:54.

Answer: 54.

2. Six distinct performers appear in a running order. A duo consists of two named performers who must appear consecutively. [4 marks]

(a) Count the running orders. [2 marks]

Treat the duo as one block among 5 units; multiply 5! by 2 internal orders.

Answer: 240.

(b) The duo also insists that one named member appears before the other. Count the orders now. [2 marks]

Only one internal order is legal, so $5! = 120$.

Answer: 120.

3. A 3-person judging panel is chosen from 8 people, including two named teachers. [5 marks]

(a) Count panels containing exactly one of the two teachers. [3 marks]

Choose the included teacher in 2 ways, then 2 from the other 6: $2 \times \binom{6}{2} = 30$.

Answer: 30.

(b) Count panels containing both teachers. [2 marks]

Fix both teachers and choose the third person from the remaining 6.

Answer: 6.

4. A design pattern is modelled by $(1+2x)^6$. [5 marks]

(a) Find the coefficient of x^2. [2 marks]

$\binom{6}{2} \times 2^2 = 15 \times 4 = 60$.

Answer: 60.

(b) Find the coefficient of x^4 and explain why it is not equal to the coefficient of x^2. [3 marks]

$\binom{6}{4} \times 2^4 = 15 \times 16 = 240$.

The factor 2 is raised to the number of x terms chosen.

Answer: 240; the binomial coefficients agree, but the powers of 2 differ.

5. Organisers survey the first 80 people who enter at 9 am to estimate the satisfaction of all 2,000 attendees. [6 marks]

(a) Identify the target population and explain two potential biases. [3 marks]

Selection is based on arrival time rather than a random frame.

Satisfaction may also change with crowding or later activities.

Answer: All 2,000 attendees; early arrivals may differ, and later attendees are excluded.

(b) Suggest a more defensible sampling approach. [3 marks]

For example, allocate samples by time blocks and randomly select within each block.

Record the frame and limitations.

Answer: Sample randomly across attendance times, with a planned selection rule and follow-up for non-response.

6. The waiting times in minutes are 2,4,4,5,5,6,7,15. [5 marks]

(a) Find the mean, median and modes. [3 marks]

Sum 48 over 8 gives 6.

Middle values are 5,5;4 and 5 each occur twice.

Answer: Mean 6; median 5; modes 4 and 5.

(b) Explain which centre you would report for a typical wait. [2 marks]

The high value pulls the mean above the main cluster.

Answer: Median 5 min, while also reporting the unusually long 15 min wait.

7. The waiting times are 2,4,4,5,5,6,7,15 minutes. Use the median-of-halves convention. [6 marks]

(a) Find Q_1, Q_3, IQR and range. [3 marks]

Lower half 2,4,4,5 has median 4.

Upper half 5,6,7,15 has median 6.5.

$6.5 - 4 = 2.5;\ 15 - 2 = 13.$

Answer: $Q_1 = 4$; $Q_3 = 6.5$; IQR 2.5; range 13.

(b) Draw a labelled min-max box plot. [3 marks]

Use a common linear minutes scale.

Mark the five-number summary $(2, 4, 5, 6.5, 15)$.

Answer: Whiskers at 2 and 15, box from 4 to 6.5, median at 5.

8. A sample of 3 weighed parcels has masses 2,5,8 kg. [6 marks]

(a) Find the mean and sample standard deviation. [3 marks]

Squared deviations $9 + 0 + 9 = 18$.

Divide by 2 and take the square root:3.

Answer: Mean 5 kg; $s = 3$ kg.

(b) A fourth parcel of 20 kg is added. Find the new mean and explain whether the old standard deviation still describes the new sample. [3 marks]

New total 35 over 4 gives 8.75.

The new squared-deviation total is 186.75, so new $s = \sqrt{\frac{186.75}{3}} \approx 7.890$ kg, much larger.

Answer: New mean 8.75 kg; the old s does not describe the new spread.

9. A normal packing model has mean 250 g and standard deviation 4 g. Packets are accepted from 244 to 258 g. Supplied $\Phi(2) = 0.97725$ and $\Phi(-1.5) = 0.06681$. [5 marks]

(a) Calculate the modelled acceptance proportion. [3 marks]

The z bounds are -1.5 and 2.

$0.97725 - 0.06681 = 0.91044$.

Answer: 0.91044.

(b) Estimate the expected rejected packets in 10,000. [2 marks]

Rejection proportion $1 - 0.91044 = 0.08956$.

Multiply by 10,000.

Answer: 895.6, approximately 896.

10. The organiser compares satisfaction scores from volunteers on an online poll with a random exit sample. [5 marks]

(a) The poll has more responses. Does that alone make it more trustworthy? Explain. [2 marks]

A larger biased voluntary sample can be less representative than a smaller random sample.

Consider who could respond and who chose to respond.

Answer: No.

(b) Both distributions are strongly left-skewed. Should an empirical-rule normal estimate be used without checking? Explain. [3 marks]

Normal symmetry is not supported.

Use observed summaries or justify another model rather than mechanically applying 68-95-99.7.

Answer: No.