Counting and statistics - Practice test 2

10 questions | 50 marks | 65 minutes

Apply restrictions, work from a frequency table and compare distributions. Normal cumulative values are supplied where needed.

Equivalent correct methods earn the corresponding marks.

1. A four-digit number is made from digits 0-6 without repetition. [5 marks]

(a) Count all such four-digit numbers. [3 marks]

The leading digit has 6 choices, then 6,5,4 choices remain.

$6 \times 6 \times 5 \times 4 = 720$.

Answer: 720.

(b) Count the numbers ending in 0. [2 marks]

Fix 0 at the end; the first 3 positions use distinct digits from 1-6.

$6 \times 5 \times 4 = 120$.

Answer: 120.

2. Six different people stand in a row; two named people must stand together. [5 marks]

(a) Count the rows. [3 marks]

Treat the named pair as one block:5! arrangements.

The pair has 2 internal orders:$5! \times 2 = 240$.

Answer: 240.

(b) Count the rows if those two must instead occupy the two ends. [2 marks]

Choose which named person is at the left in 2 ways.

Arrange the other 4 in $4!$ ways: $2 \times 24 = 48$.

Answer: 48.

3. A panel of 3 is chosen from 6 adults and 4 students. [5 marks]

(a) Count panels with exactly 2 adults. [2 marks]

$\binom{6}{2}\binom{4}{1} = 15 \times 4 = 60.$

Answer: 60.

(b) Count panels with at least 1 student. [3 marks]

$\binom{10}{3} - \binom{6}{3} = 120 - 20 = 100.$

Answer: 100.

4. Consider $(2x-1)^5$. [5 marks]

(a) Find the coefficient of x^3. [3 marks]

Choose 3 brackets contributing $2x$: $\binom{5}{3} \times 2^3 \times (-1)^2 = 80.$

Answer: 80.

(b) Find the sum of all coefficients. [2 marks]

Substitute $x = 1$: $(2-1)^5 = 1.$

Answer: 1.

5. A school has 420 juniors and 280 seniors. It wants a sample of 50. [5 marks]

(a) Allocate a proportional stratified sample. [2 marks]

$\frac{420}{700} \times 50 = 30;\ \frac{280}{700} \times 50 = 20.$

Answer: 30 juniors and 20 seniors.

(b) Describe a valid selection method and one possible source of remaining bias. [3 marks]

Use enrolment lists as sampling frames.

Follow up consistently and report missing responses.

Answer: Randomly select within each group; non-response may still cause bias.

6. The table records the number of library visits per week in a sample. [5 marks]

Visits	0	1	2	3
Frequency	2	3	4	1

(a) Find the sample size and mean number of visits. [3 marks]

Frequency total $= 2 + 3 + 4 + 1 = 10$.

Weighted total $= 0 \times 2 + 1 \times 3 + 2 \times 4 + 3 \times 1 = 14$; mean $\frac{14}{10} = 1.4$.

Answer: $n = 10$; mean 1.4 visits.

(b) Find the median and mode. [2 marks]

The fifth observation is 1 and the sixth is 2.

The most frequent value is 2.

Answer: Median 1.5; mode 2.

7. Group A has five-number summary $(4, 8, 12, 16, 24)$; Group B has $(6, 10, 15, 22, 30)$. [5 marks]

(a) Compare medians and IQRs with numerical evidence. [3 marks]

A $\text{IQR} = 16 - 8 = 8$; B $\text{IQR} = 22 - 10 = 12$.

Answer: B has a 3-unit higher median and a 4-unit larger IQR.

(b) Does the summary prove every B value exceeds every A value? Explain. [2 marks]

The intervals overlap and the summaries do not list each observation.

Answer: No.

8. A sample contains 2,2,6,6. [5 marks]

(a) Calculate its mean and sample standard deviation to 3 decimal places. [3 marks]

Squared deviations total 16.

$s = \sqrt{\frac{16}{3}} = 2.309401\ldots$

Answer: Mean 4; $s \approx 2.309$.

(b) The readings are in metres. What unit belongs to the variance? [2 marks]

Squaring deviations squares the measurement unit.

Answer: Square metres.

9. A normal model has mean 100 and standard deviation 10. Supplied values: $\Phi(1.5) = 0.93319$, $\Phi(-0.5) = 0.30854$. [5 marks]

(a) Find $P(95 < X < 115)$. [3 marks]

Standardized bounds are -0.5 and 1.5.

$0.93319 - 0.30854 = 0.62465.$

Answer: 0.62465.

(b) Find the expected number in that interval among 800 items. [2 marks]

$800 \times 0.62465 = 499.72.$

Answer: 499.72, approximately 500 items.

10. A company models completion time as normal with mean 40 min and sd 6 min. The standard-normal 90th percentile is 1.28155. [5 marks]

(a) Find the model's 90th-percentile completion time. [3 marks]

$40 + 6 \times 1.28155 = 47.6893.$

Answer: Approximately 47.69 min.

(b) A manager says every job must finish within that time. Evaluate the claim. [2 marks]

A 90th percentile is not a maximum and model assumptions may also be imperfect.

Answer: Incorrect; the model puts about 10% above that time.