Counting and statistics - Practice test 1

10 questions | 45 marks | 60 minutes

Begin with counting choices, then analyse a small survey and an introductory normal model. Show formulas and explain your choices.

Equivalent correct methods earn the corresponding marks.

1. A lunch menu offers 4 mains,3 sides and 2 drinks. Every combination is available. [4 marks]

(a) Count the complete meals. [2 marks]

$4 \times 3 \times 2 = 24$.

Answer: 24.

(b) One main is unavailable. Count the remaining complete meals. [2 marks]

$3 \times 3 \times 2 = 18$.

Answer: 18.

2. Five different posters are displayed in a row. [4 marks]

(a) Count all possible rows. [2 marks]

$5! = 120$.

Answer: 120.

(b) A particular poster must be at the left end. Count the rows. [2 marks]

Fix that poster, then arrange the other 4 in $4! = 24$ ways.

Answer: 24.

3. A group of 9 students chooses a captain, deputy and treasurer, all different people. [4 marks]

(a) Explain whether order matters. [2 marks]

Changing which person holds which role changes the outcome.

Answer: Yes: the three roles are distinct.

(b) Count the assignments. [2 marks]

${}^{9}P_{3} = 9 \times 8 \times 7 = 504.$

Answer: 504.

4. A library chooses 3 books from 8 different novels. [4 marks]

(a) Count the selections if no reading order is specified. [2 marks]

$\binom{8}{3} = \frac{8!}{3!\,5!} = 56.$

Answer: 56.

(b) Count the selections containing one specified novel. [2 marks]

Include the specified book and choose 2 of the remaining 7: $\binom{7}{2} = 21.$

Answer: 21.

5. Consider the expansion of $(x+3)^4$. [5 marks]

(a) Write the five Pascal coefficients. [2 marks]

Each interior coefficient is the sum of the two above it.

Answer: 1,4,6,4,1.

(b) Expand the expression. [3 marks]

Use descending powers of x and ascending powers of 3.

The terms are $x^4, 4x^3(3), 6x^2(9), 4x(27), 81.$

Answer: $x^4 + 12x^3 + 54x^2 + 108x + 81.$

6. A teacher randomly selects 30 of 300 students and records journey time and number of bus transfers. [4 marks]

(a) Identify the population and sample. [2 marks]

The target is all 300; only 30 are observed.

Answer: Population:300 students; sample:the 30 selected.

(b) Classify both recorded variables. [2 marks]

Time is measured; transfers are counted.

Answer: Journey time is continuous numerical; number of transfers is discrete numerical.

7. Five observed daily sales totals are 3,4,4,5,14. [5 marks]

(a) Find the mean, median and mode. [3 marks]

The sum is 30, so $\text{mean} = \frac{30}{5} = 6$.

The middle value and most frequent value are 4.

Answer: Mean 6; median 4; mode 4.

(b) Which centre better describes an ordinary day in this sample? Explain. [2 marks]

The unusually large 14 pulls the mean above the cluster 3-5.

Answer: Median 4.

8. The ordered data are 2,3,5,7,8,10,14. Use the median-of-halves convention, excluding the middle value. [5 marks]

(a) Find the five-number summary. [3 marks]

Median 7; lower half 2,3,5 has median 3; upper half 8,10,14 has median 10.

Answer: $(2, 3, 7, 10, 14)$.

(b) Find the IQR and range. [2 marks]

$10 - 3 = 7; 14 - 2 = 12$.

Answer: IQR 7; range 12.

9. A sample is 4,6,8. [5 marks]

(a) Calculate the sample mean and sample standard deviation. [3 marks]

Squared deviations are 4,0,4, total 8.

Sample variance $= \frac{8}{3-1} = 4$; $s = 2$.

Answer: Mean 6; $s = 2$.

(b) Every reading increases by 10. Find the new mean and s. [2 marks]

A translation changes the centre but not deviations or spread.

Answer: Mean 16; $s = 2$.

10. Bottle volumes follow an approximately normal model with mean 500 mL and standard deviation 5 mL. [5 marks]

(a) Estimate the percentage between 490 and 510 mL using the empirical rule. [2 marks]

The bounds are mean ± 2 standard deviations.

Answer: About 95%.

(b) Estimate the expected number above 510 mL among 4,000 bottles. Explain the word expected. [3 marks]

The upper tail is about 2.5%.

$4000 \times 0.025 = 100$; actual counts can vary.

Answer: About 100 bottles; it is an average prediction, not a guaranteed batch count.