Introduction to calculus - Topic test 5

Name: _____ Date: _____

10 questions | 56 marks | 75 minutes

A harder synthesis paper with exact tangent conditions, parameters, weighted constraints and model evaluation.

A scientific calculator may be used.

1. Let $f(x) = x^3 - 3x$. [4 marks]

(a) Find its average rate on $[-1, 1]$. [2 marks]

(b) Compare with its instantaneous rate at 0. [2 marks]

2. A quadratic has form $f(x) = x^2 + bx + c$. [6 marks]

(a) Find the difference quotient at input a and its limit. [3 marks]

(b) If $f'(3) = 0$ and $f(1) = 5$, find b and c. [3 marks]

3. Find tangents to $f(x) = x^3 - 3x$ parallel to $3x - y + 2 = 0$. [6 marks]

(a) Find the possible tangent inputs. [3 marks]

(b) Give both exact tangent equations. [3 marks]

4. $f(x) = x^3 - 3kx$ has a stationary point at $x = 2$. [5 marks]

(a) Find k and all stationary coordinates. [3 marks]

(b) Classify them. [2 marks]

5. A model cost is $C(x) = 2x^2 - 16x + 50$ for $1 \leq x \leq 6$. [4 marks]

(a) Find its stationary value. [2 marks]

(b) Use all relevant candidates to justify the absolute minimum. [2 marks]

6. A garden against a wall has two perpendicular sides costing 2 dollars per metre and its remaining parallel side costing 3 dollars per metre. Budget is 120 dollars. Let the perpendicular side be x and parallel side y. [7 marks]

(a) Form the budget constraint and area function. [3 marks]

(b) Maximise the area. [4 marks]

7. A particle follows $s(t) = t^3 - 6t^2 + 9t$ metres on $0 \le t \le 4$. [6 marks]

(a) When does it move in the negative direction, and what is its speed at $t = 2$? [3 marks]

(b) Find its total distance. [3 marks]

8. $f(x) = x^3 - ax + b$ touches the x-axis with a horizontal tangent at $x = 2$. [6 marks]

(a) Find a and b. [3 marks]

(b) Find the other stationary point and classify both. [3 marks]

9. Positive numbers a and b satisfy $a+b=18$. Maximise the product ab^2. [6 marks]

(a) Form an objective and state its domain. [2 marks]

(b) Find the greatest product and the corresponding numbers. [4 marks]

10. Consider $M(x)=-x^3+12x^2-36x+50$ on $[0,8]$. [6 marks]

(a) Find all stationary points and classify them. [3 marks]

(b) Find every input giving the global maximum and explain why a local maximum alone is insufficient. [3 marks]