Introduction to calculus - Topic test 4

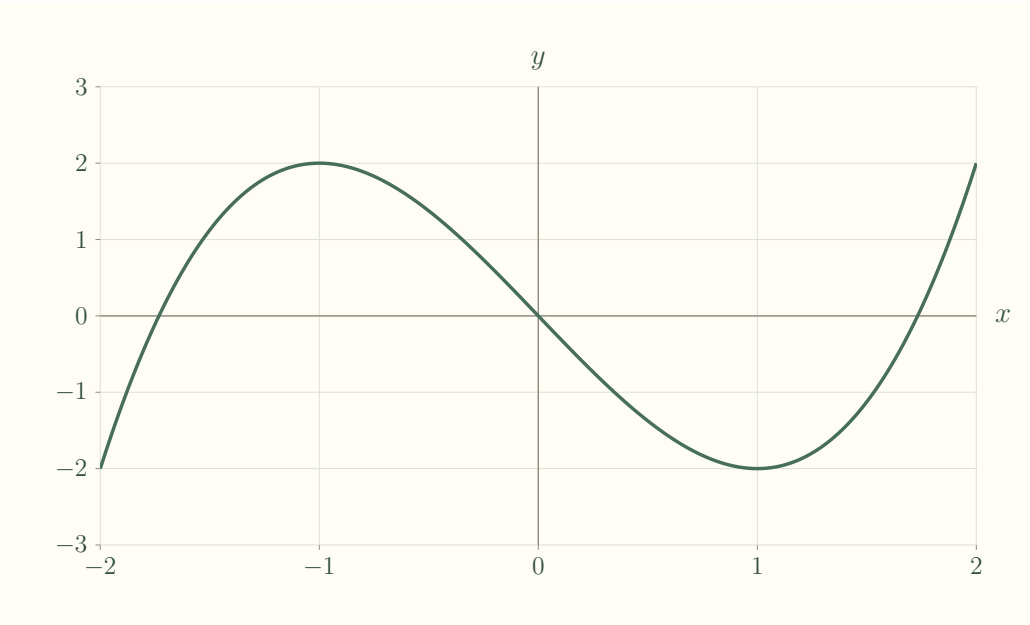
Name: _____ Date: _____

10 questions | 49 marks | 65 minutes

Analyse different representations, tangent conditions, changes of direction and the validity of solutions.

A scientific calculator may be used.

1. The supplied graph represents $f(x) = x^3 - 3x$. [5 marks]



(a) Read the approximate stationary coordinates from the graph. [2 marks]

(b) Verify them exactly with a derivative. [3 marks]

2. Compare $f(x) = x^2$ and $g(x) = 2x^2 + 3$. [4 marks]

(a) Find both derivative functions. [2 marks]

(b) Based on this comparison, conjecture the derivative of $af(x) + c$ for constants a, c and a differentiable function f. Does this one comparison prove your general rule? [2 marks]

3. $f(x) = x^3 + kx$. Its tangent at $x = 1$ has gradient 5. [4 marks]

(a) Find k. [2 marks]

(b) Find the tangent equation at that point. [2 marks]

4. For $f(x) = x^3 - x$ at the origin, [4 marks]

(a) find the tangent equation. [2 marks]

(b) find the normal and verify perpendicularity. [2 marks]

5. Analyse $f(x) = x(x-2)^2$. [6 marks]

(a) Find intercepts and stationary coordinates. [4 marks]

(b) Classify the stationary points. [2 marks]

6. $s(t) = t^3 - 9t^2 + 24t$ metres for $0 \leq t \leq 6$. [6 marks]

(a) Find rest times and total distance. [4 marks]

(b) At $t = 3$, find acceleration and decide whether the particle is at rest. [2 marks]

7. For $C(x) = x^2 + 2x + 3$, [4 marks]

(a) find its unrestricted minimum. [2 marks]

(b) find its minimum if only $0 \leq x \leq 4$ is allowed. [2 marks]

8. A rectangular enclosure has a single internal divider parallel to its width. Total fencing is 60m. Let width be x and length y. [6 marks]

(a) Form the area in terms of x alone. [3 marks]

(b) Find the greatest area and dimensions. [3 marks]

9. $s(t) = t^3 - 6t^2 + 12t$ metres on $0 \le t \le 4$ seconds. [5 marks]

(a) Show the particle is at rest once inside the interval, without turning. [3 marks]

(b) Find displacement and total distance. [2 marks]

10. A student uses $A = x(20 - x)$ for a rectangle and writes $A' = 20 - x$, obtaining $x = 20$. [5 marks]

(a) Correct the derivative and the stationary input. [2 marks]

(b) Explain the geometric fault in the proposed $x = 20$ answer. [3 marks]