Introduction to calculus - Topic test 3

Name: _____ Date: _____

10 questions | 52 marks | 65 minutes

Select and evaluate constrained models, including non-turning stationary points and boundary optima.

A scientific calculator may be used.

1. A production cost is $C(n) = 100 + 3n + 0.02n^2$ dollars, for a continuous model variable n. [5 marks]

(a) Find the average cost increase per unit from $n = 10$ to $n = 30$. [3 marks]

(b) Find $C'(20)$ and compare the two rates. [2 marks]

2. Let $f(x) = (x - 3)^2$. [4 marks]

(a) Expand f and find its derivative. [2 marks]

(b) Explain why its tangent is horizontal at its minimum. [2 marks]

3. A tangent to $y = x^2 + 4$ touches at input a and passes through the origin. [6 marks]

(a) Express its equation in terms of a. [3 marks]

(b) Find both possible tangents. [3 marks]

4. Consider $f(x) = x^3 - 6x^2 + 12x$ on $[0, 4]$. [5 marks]

(a) Find its stationary point and decide whether it is a turn. [3 marks]

(b) Find its greatest and least values on the interval. [2 marks]

5. A restricted cost model is $C(x) = x^2 - 8x + 30$ for $5 \leq x \leq 10$. [4 marks]

(a) Find the unrestricted stationary input and assess its relevance. [2 marks]

(b) Find the permitted minimum. [2 marks]

6. Squares of side x cm are cut from the corners of an 18cm square, which is folded into an open box. [6 marks]

(a) Write its volume and physical domain. [2 marks]

(b) Find and justify its maximum volume. [4 marks]

7. A profit model is $P(x) = -x^3 + 9x^2 - 15x$ on $[0, 6]$. [6 marks]

(a) Find its stationary coordinates and types. [4 marks]

(b) Find the global maximum on the interval. [2 marks]

8. A vertical height model is $h(t) = -5t^2 + 20t + 1$ metres after t seconds, from launch until the object first reaches ground level. Take upwards as positive. [5 marks]

(a) Find the maximum height and when it is reached. [3 marks]

(b) Find acceleration and interpret its sign. [2 marks]

9. The price per item is $p = 30 - 0.5x$ dollars and cost is $C = 5x + 20$, for $0 \leq x \leq 50$. [6 marks]

(a) Form the profit function. [2 marks]

(b) Find the profit-maximising quantity, price and profit. [4 marks]

10. A derivative is $f'(x) = (x+1)^2(x-2)$. [5 marks]

(a) Determine its sign intervals. [3 marks]

(b) Classify the stationary inputs. [2 marks]